

Numerical Investigation of Entropy Generation in a C-shaped Channel for Laminar Non-Newtonian Fluid Under a Constant Heat Flux

Naas Toufik Tayeb¹, Lasbet Yahia¹, Benzaoui Ahmed² & Loubar Khaled³

¹Department of Mechanical Engineering, Laboratoire de développement en mécanique et Matériaux, Ziane Achour University, Djelfa 17000, Algeria

²Laboratoire Thermodynamique et Systèmes Energétiques (LTSE), University of Houari Boumediene, Algeria

³Ecole des Mines de Nantes. Nantes 44307, Cedex 3, France

Corresponding author: toufiknaas@gmail.com

Abstract: A steady, three-dimensional numerical analysis of non-Newtonian laminar flow and heat transfer in complex geometry has been performed. The Navier-Stokes equations and constant wall heat flux boundary conditions are adopted. The solutions are presented to a various inlet generalized Reynolds number (Re_g) at fixed cross section area for the C-shaped channel.

The results show that the entropy generation number and non-dimensional entropy generation number are less for the Non-Newtonian fluid than for the Newtonian fluid for the C-shaped channel when aniline is heated. However, the effects of generalized Reynolds number (Re_g) and flow behavior index (n) on the entropy generation have also been studied to provide the heat transfer performance.

Keywords: C-shaped channel, Laminar flow, heat transfer, non-Newtonian fluids, entropy generation.

1 Introduction

Non-Newtonian fluids flow in complex geometries are widely used in various engineering applications [1-2]. Consequently, extensive, experimental and analytical studies have been carried out on such flow systems. They have better characteristics of thermally and developing flow.

Mohammad et al [3] investigate the entropy generation in a helically coiled tube in laminar flow under a constant heat flux. The results showed the effect of different flow conditions such as mass velocity, inlet vapor quality, saturation temperature, and heat flux on contributions of pressure drop and heat transfer in entropy generation. For implementing the calculation of entropy generation into CFD (Computational Fluid Dynamics) codes, Koand Ting [4-5] analyzed the entropy production in incompressible laminar shear flows in heated curved rectangular duct. Their results reveal that the addition of rib can effectively reduce the entropy generation from heat transfer irreversibility.

As for the entropy analysis and the optimal designs based on minimal entropy generation principle in non-Newtonian fluid flows, the relevant studies are relatively sparse. An analytical study of entropy generation for fully developed non-Newtonian flow through microchannels, in which the effects of viscous dissipation on the entropy production were investigated by Hung [6]. Vishal [7] investigated the viscous dissipation effect on entropy generation for non-Newtonian fluids in laminar fluid flow through a microchannels subjected to constant heat flux. In a recent study of Kefayati [8], heat

transfer and entropy generation on laminar natural convection of non-Newtonian nano-fluids by Finite Difference Lattice Boltzmann Method.

In the present study, our main aim is to investigate numerically the difference between the entropy generations caused by heat transfer and fluid friction, and the effects of generalized Reynolds number on the entropy generation for laminar flow in C-shaped channel under wall heat flux conditions.

2 Physical model

Figure 1 show the C-Shaped channel analyzed in the present paper, which has been introduced by Liu et al [1] and generated spatially chaotic flows by Beebe [9]. The channel cross-section is square (0.015 m × 0.015 m). The hydraulic diameter D_h is 0.015 m. The unfolded length is equal to 0.135 m.

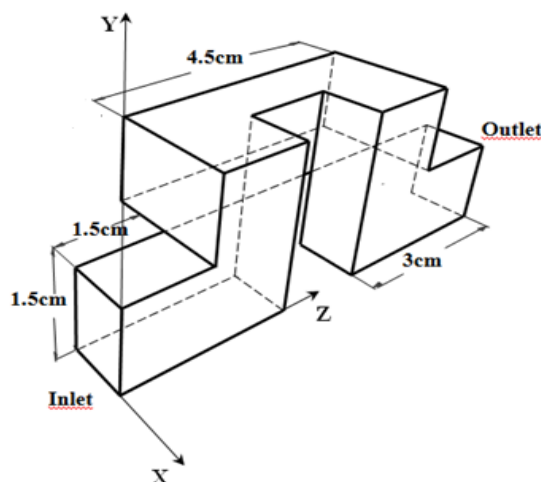


Figure 1: Schematic representation of the C-shaped channel

3 Analysis

3.1 Non-Newtonian fluid model

In order to model the flow of non-Newtonian fluids, the purely viscous (i.e., inelastic) non-Newtonian character of the fluid that is studied here is represented by Bird et al [10] power-law model for the case of both shear-thinning and Newtonian fluids. The constitutive relation between the shear stress τ (Pa) and the shear rate γ (s⁻¹) can be described by a simple power law expression:

$$\tau = k\gamma^n \quad (1)$$

Where k (Pa·sⁿ) is power-law consistency index and n is the flow behavior index of the fluid.

The nonlinear relationship between the apparent viscosity μ_{app} (Ns m⁻²) and the shear rate γ is given by the constitutive equation:

$$\mu_{app} = k\gamma^{n-1} \quad (2)$$

We will consider five different flow behavior indexes that are associated with shear-thinning ($n = 0.5, 0.6, 0.7, 0.8$ and 0.9) and Newtonian ($n = 1$) fluids. The consistency index (k) is adapted in each non-Newtonian case in order to give the same generalized Reynolds number as it was considered for the Newtonian flow ($Re = \rho U_i D_h^n / \mu$). This generalized Reynolds number (Re_g), can be written for the power-law fluid as [11]:

$$Re_g = \frac{\rho U_i^{2-n} D_h^n}{\left[8^{n-1} \left(b^* + \frac{a^*}{n} \right)^n k \right]} \quad (3)$$

Where, a^* and b^* equal 0.2121 and 0.6771 respectively, for square channel, ρ density of fluid (kg m^{-3}), and U_i (m/s) is the inlet velocity. The non-dimensional wall heat flux q^* is defined according to the external wall heat flux, \dot{q} (W/m^2), as: $q^* = \dot{q} D_H / \lambda T_i$. Where T_i is the inlet temperature of fluid and λ is the thermal conductivity (W/mK).

3.2 Entropy generation in laminar flows

Based on the temperature and velocity distribution of the flow field, the local entropy generation in the laminar flow is given for three dimensional flow as follows [12,14]:

$$S_{gen}''' = \frac{\lambda}{T^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 \right] + \frac{\mu_{app}}{T} \left[2 \left(\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right) + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right] \quad (4)$$

The first term on the right-hand side is due to heat transfer and the second term is due to friction factor.

The total volumetric entropy generation is:

$$S_{gen}''' = S_{gen,\Delta T}''' + S_{gen,\Delta P}''' \quad (5)$$

The non-dimensional entropy generation numbers are defined as follows:

$$N_{S,\Delta T} = \frac{\iiint_V S_{gen,\Delta T}''' dV}{\dot{Q}/T_i} \quad (6)$$

$$N_{S,\Delta P} = \frac{\iiint_V S_{gen,\Delta P}''' dV}{\dot{Q}/T_i} \quad (7)$$

$$N_S = \frac{\iiint_V S_g''' dV}{\dot{Q}/T_i} \quad (8)$$

Where V is total volume of the channel and \dot{Q} is the total heat transfer rate.

Bejan number, which is defined as the ratio of the heat transfer entropy generation to the global entropy generation due to heat transfer and fluid friction, can be expressed as follows:

$$Be = \frac{S_{gen,\Delta T}'''}{S_{gen,\Delta T}''' + S_{gen,\Delta P}'''} \quad (9)$$

The value of Be ranges from 0 to 1. Accordingly, $Be = 0$ and $Be = 1$ are two limiting cases representing the irreversibility is dominated by fluid friction and heat transfer, respectively.

4 Numerical method and grid independent test

The three-dimensional system is established for C-shaped channel using the commercial CFD software. Navier-Stokes and energy equations are discretized by finite volume method to calculate the thermodynamic characteristics of Newtonian and non-Newtonian fluids. The momentum and energy terms are treated with the second order upwind scheme, while the pressure term takes the standard scheme. The gradients on the faces are evaluated with the Node-Based method, i.e., taking the arithmetic average of the nodal values on the face. The double-precision segregated solver is adopted for the computation, and pressure and velocity are coupled with the 'SIMPLE' algorithm. The computations were considered to be converged once all the scaled residuals are less than 10^{-7} and the global imbalances, representing overall conservation don't exceed 10^{-5} .

To optimise the mesh size it was necessary to carry out a mesh independence study; this was done by performing a number of simulations with different mesh sizes, starting from a coarse mesh and refining it until results were no more dependent on the mesh size. Therefore, the grids are ranging from 30 to 60 nodes in the x, y and z directions for each 0.015 m. The results are presented below for the case of non-Newtonian fluid flow in the C-shaped geometry considering a steady laminar flow, at fixed generalized Reynolds number $Re_g = 200$ and Power-law index $n = 0.5$.

Figure 2 shows the evolutions entropy generation due to heat transfer and entropy generation due to friction factor for various grids. It indicates that the entropy generation is sensitive to the grid mesh except for the mesh densities (50*50*50) and (60*60*60) where no significant difference is seen. As consequence, the (50*50*50) grid is chosen as the optimal grid mesh for the computations.

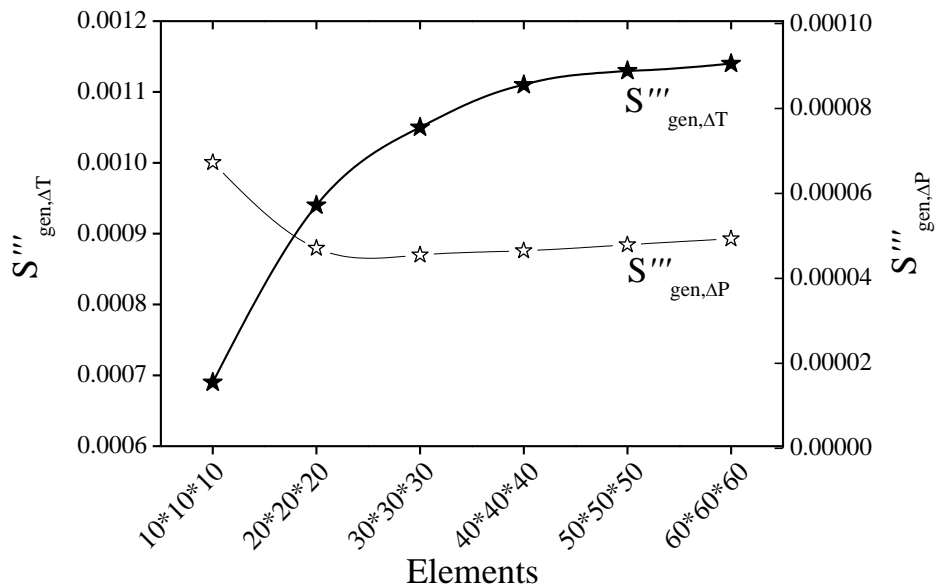


Figure 2: entropy generation due to heat transfer and friction factor for different elements in C-shaped channel, for $n = 0.5$

5 Results and discussions

Numerical solutions have been investigated for laminar flow of non-Newtonian fluid through the comparison of the entropy generation due to heat transfer with the entropy generation due to friction factor in C-shaped channel.

For understanding the development of entropy generation in the C-shaped channel, effect of generalized Reynolds number on entropy generation due to heat transfer and friction factor, with different value of n , are shown in figure 3 and 4, respectively. The results are presented for the cases of $n = 0.5$ to 0.9 and $q^* = 0.407$. A clear development can be found from the figure that $S'''_{gen,\Delta T}$ in all cases of non-Newtonian fluids is increases as the flow behavior index n decreases. As flow behavior index becomes smaller, increases more, which reveal again that the effect of the generalized Reynolds number on heat transfer performance is substantial. Obviously, at $n = 0.9$, $S'''_{gen,\Delta P}$ is much larger than $n < 0.9$, indicating the entropy generation under the current flow condition is dominated by the fluid frictional irreversibility,

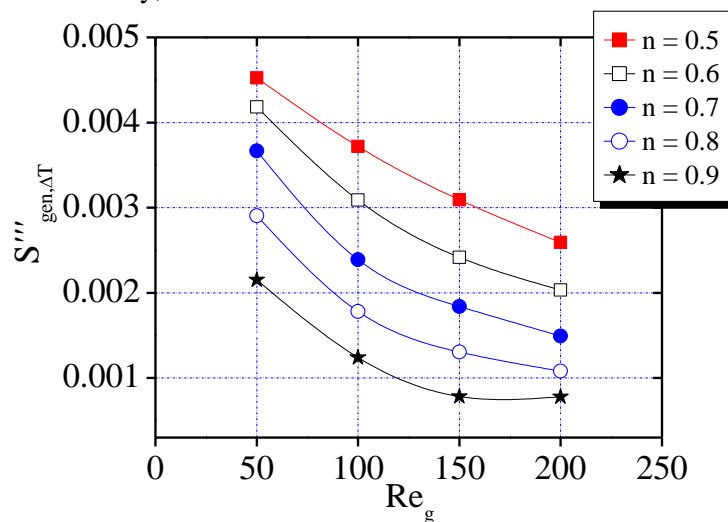


Figure 3: Evolutions of the entropy generation rate due to heat transfer ($\dot{S}_{gen,\Delta T}$) for different Power-law indexes with various values of Re_g .

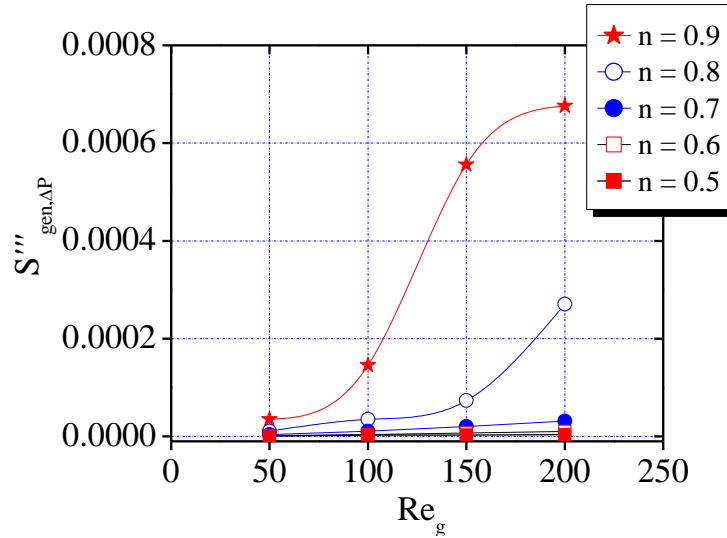


Figure 4: Evolutions of the entropy generation rate due to friction factor ($\dot{S}_{gen,\Delta P}$) for different Power-law indexes with various values of Re_g .

The variation of the global entropy generation \dot{S}_{gen} and the global non-dimensional entropy generation $N\dot{S}_{gen}$ with the flow behavior index n for different values of Re_g has been shown in figures 5 and 6, respectively. For low Re_g , the global non-dimensional entropy generation $N\dot{S}_{gen}$ increases with decreasing flow behavior index n . The reason behind this trend is the fact that the temperature gradients inside the flow increase with decrease in flow behavior index, which leads to a decline in entropy generated due to friction factor. For high values of Re_g , entropy generation under the current flow condition is dominated by the fluid friction factor irreversibility, which is due to larger gradients in velocity at walls. Therefore, the fluid system with less n and Re_g would result in better working performance.

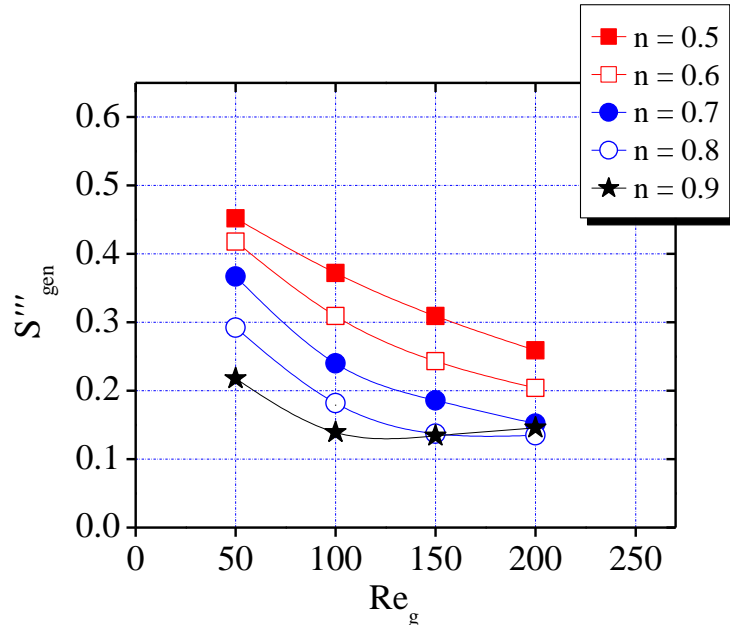


Figure 5: Evolutions of the global entropy generation rate \dot{S}_{gen} for different Power-law indexes with various generalized Reynolds number.

Figure 7 illustrates the effect of Re_g on the global Bejan number. It can be seen the values of Be for non-Newtonian fluid cases are much larger than 0.5. Therefore, the contribution to the global entropy

generation is mainly due to heat transfer irreversibility while the contribution of friction factor irreversibility is minor. On the contrary, for lower values of n , Bejan number indicates that the maximum entropy produced is mainly due to the fluid friction irreversibility. This is because the velocity gradients are higher than temperature gradients. Therefore, the fluid friction irreversibility is less prominent, leading to higher Bejan number for all cases of non-Newtonian fluids.

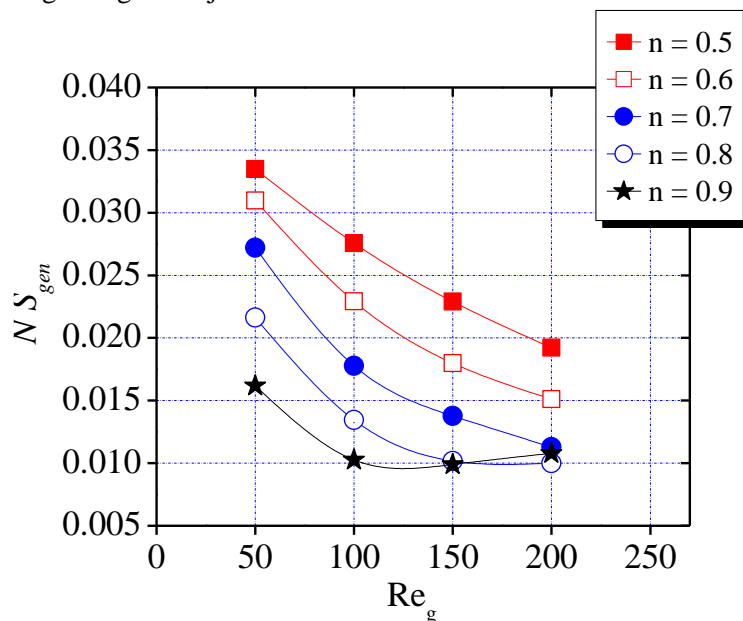


Figure 6: Evolutions of the global non-dimensional entropy generation rate NS_{gen} for different Power-law indexes with various generalized Reynolds number.

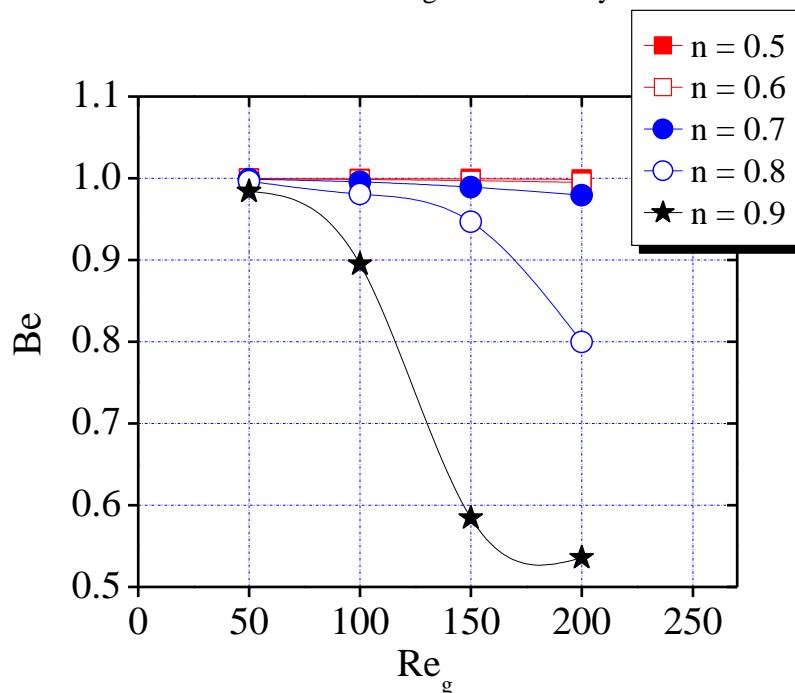


Figure 7: Evolutions of the global Bejan number (Be) for different Power-law indexes with various generalized Reynolds number.

6 Conclusion

This paper presents numerical study of the second law of thermodynamics to Newtonian and non-Newtonian fluids flow flowing in chaotic geometry, with heat flux boundary condition, while the entropy generated from heat transfer irreversibility and frictional irreversibility is analyzed separately in detail.

This work has been performed for the important parameters in the following ranges: generalized Reynolds number ($Re_g = 50$ to 200), flow behavior index ($n = 0.5$ to 0.9) and non-dimensional heat flux ($q^* = 0.407$). Some results are summarized as follows:

- The increase in generalized Reynolds number causes the reduction in heat transfer contribution and the increase in the pressure drop one, that's mean the entropy generation due to heat transfer decreases while the entropy generation due to friction factor augments.
- For the all cases of non-Newtonian fluids. The increase in the value of flow behavior index shows the contribution of heat transfer decreases whereas the pressure drop one increases. Consequently, the global entropy generation decreases.
- The increase in the value of generalized Reynolds number at constant external heat flux causes Bejan number to declines.

Nomenclature

a^*	geometric constant in generalized Reynolds number, Eq (3)
b^*	geometric constant in generalized Reynolds number, Eq (3)
Be	Bejan number
D_h	Hydraulic diameter, (m)
K	power-law consistency index, (Pa/s)
n	flow behavior index
N_s	Non-dimensional entropy generation
$N_{S,\Delta T}$	Non-dimensional entropy generation due to heat transfer
$N_{S,\Delta P}$	Non-dimensional entropy generation due to heat transfer
p	Pressure, (Pa)
q^*	The non-dimensional external heat flux
q''	external heat flux, (W/m^2)
\dot{Q}	heat transfer rate, (W)
Re	Reynolds number
Re_g	generalized Reynolds number
\dot{S}_g	global entropy generation, (W/K)
$\dot{S}_{g,\Delta T}$	entropy generation due to heat transfer, (W/K)
$\dot{S}_{g,\Delta P}$	entropy generation due to friction factor, (W/K)
T	temperature, (K)
T_i	inlet temperature, (K)
u	velocity, (m/s)
U_i	inlet velocity, (m/s)
V	volume, (m^3)
μ	viscosity, ($N\ s/m^2$)
μ_{app}	apparent viscosity, (Ns/m^2)
$\dot{\gamma}$	shear rate, (1/s)
ρ	density, (Kg/m^3)
λ	thermal conductivity, $W/(m\ K)$

References

- [1] H. Aref. Stirring by chaotic advection. J. Fluid Mech, 143-121.1984.
- [2] H. Aref. Order in chaos. Nature, 401, 756-757.1999.
- [3] Mohammad Ali Abdous, Hamid Saffari, Hasan Barzegar Avval, Mohsen Khoshzat "Investigation of entropy generation in a helically coiled tube in flow boiling condition under a constant heat flux", international journal of refrigeration 60, 2015, 217–233.
- [4] T.H. Ko, "Numerical analysis of entropy generation and optimal Reynolds number for

- developing laminar forced convection in double-sine ducts with various aspect ratios”, *International Journal of Mass and Heat Transfer* 49 (3-4), 2006, 718–726.
- [5] T.H. Ko, K. Ting, “Entropy generation and optimal analysis for laminar forced convection in curved rectangular ducts: a numerical study”, *International Journal of Thermal Sciences* 45 (2), 2006, 138–150.
- [6] Y.M. Hung “Viscous dissipation effect on entropy generation for non-Newtonian fluids in microchannels”, *International Communications in Heat and Mass Transfer* 35, 2008, 1125–1129.
- [7] Vishal Anand “ Slip law effects on heat transfer and entropy generation of pressure driven flow of a power law fluid in a microchannel under uniform heat flux boundary condition”, *Energy* 76, 2014, 716e732.
- [8] G.H.R. Kefayati “Simulation of heat transfer and entropy generation of MHD natural convection of non-Newtonian nanofluid in an enclosure” , *International Journal of Heat and Mass Transfer* 92, 2016, 1066–1089.
- [9] D. Beebe, R. Adrian, M. Olsen, M. Stremmer, H. Aref, B. Jo, “Passive mixing in microchannels: fabrication and flow experiments”, *Mécanique Ind.* 2, 2001, 343–348.
- [10] R.B. Bird. W.E. Stewart. and E.N. Lightfoot.. *Transport Phenomena*. Seconded. Wiley, New York 2002.
- [11] A. B. Metzner and J. C. Reed, “Flow of Non-Newtonian Fluids -Correlation of the Laminar Transition, and Turbulent-Flow Regions”, 1955,*AIChE J.* 1: 434.
- [12] A. Bejan, *Entropy Generation Through Heat and Fluid Flow*, Wiley, New York, 1982.
- [13] P.K. Nag, N. Kumar, “Second law optimization of convection heat transfer through a duct with constant heat flux”, *Int. J. Energy Res.* 13,189, 537–543.
- [14] A. Bejan, “Entropy Generation Minimization”, CRC Press, Boca Raton, FL, 1996.