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Effect of Bubbles Number on Cavitating Flow Through a Venturi

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Abstract: The motion of the bubble within a liquid-gas flow through a Venturi nozzle is investigated using a set of coupled nonlinear ordinary differential equation. The dynamics of a bubble is modeled by the use of the mass and momentum phase equations which are coupled with the Rayleigh-Plesset equation of the N bubbles dynamics. However, assuming that same initial condition for identical and equidistance between bubbles. The obtained numerical result shown that, as number of bubbles is increased the natural frequency and the damping of the bubbles decrease. Beside, the distance between bubbles decrease leads to increase of the damping and the natural frequency.

Keywords: Bubbly flow, Venturi meter, tow phase flow, Cavitations

1 Introduction

It is well known that the Venturi is a robust method for measuring the flow of a single-phase fluid for high Reynolds numbers. Multiphase flow measuring is generally more difficult. The density of a gas-liquid mixture depends upon the volume fraction of the gas, and the phases densities. The velocity of the gas within the Venturi is likely to differ from that of the liquid. The investigations of homogeneous steady-state cavitating nozzle flows, using spherical bubble dynamics with a polytropic thermal process [1], have shown some flow instabilities illustrated by flashing flow phenomenon.

The flow model, a generally used, is a nonlinear continuum bubbly mixture which is coupled with the dynamics equation of the bubble. A three equations model was first proposed by van Wijngaarden [2 ,3] and has been used for studying steady and transient shock wave propagation in bubbly liquids, by omitting the acceleration of the mean flow. This model has been also considered by Wang and Brennen [1], in the case of converging-diverging nozzle, with an upstream variable void fraction. It was observed that significant change of the flow characteristics depends strongly on the latter and a critical bubbles radius have been obtained. Considering the gas nucleation rate, as a source term in the mass conservation equation of the bubbles, Delale et al [4] have used the previous model for the same converging-diverging nozzle. They have concluded that the encountered flow instability can be stabilised by thermal damping. Several authors have also considered the bubble dynamics equation under an appropriate form to the choose example. Among these, Wang and Brennen [5] have written it in time and radial coordinate, for a bubbly mixture, where the shock wave have been studied for spherical cloud of cavitating bubbles. Besides effect of the shocks on the bubbles interactions have been also analysed. The same Rayleigh-Plesset equation has been used by Gaston et al [6] by modelling the bubble as a potential source. The stream function has been written in function of spatial coordinate and the source term. They have analysed the effect of complex interactions through a Venturi. By introducing liquid quantity and motion equation in a spatial Rayleigh-Plesset dynamics relation, Moholkar and Pandit [7] have obtained a global dynamic equation witch have been resolved by in a three steps method. In their work they have studied the effect of the downstream pressure, the Venturi pipe ratio, the initial bubble size an the upstream void fraction, on the dynamics of the flow. Considering a one bubble motion in a Venturi, Soubiran and Sherwood [8] have obtained a dynamic equation of the flow, based on the, acting different force.

A. Ooi and R. Manasseh [9] have studied coupling effects on acoustic signature from non-linear oscillations of a group of micro bubbles by the use of Rayleigh-Plesset equation, where bubbles number and their natural frequency are significantly dependant.

More recently, Ashrafizadeh and Ghassemi [10] have experimentally and numerically investigate the effect of the geometrical parameters, such as throat diameter, throat length, and diffuser angle, on the mass flow rate, critical pressure ratio and application rang of small-sized cavitating venturis (CVs). The obtained results show that the CVs in very small size are also capable in controlling and regulating the mass flow rate while their characteristic curves are similar to those of ordinary CVs with larger throat sizes. Also, by decreasing the throat diameter of CVs, the choked mode region, critical pressure and discharge coefficient decrease. By decreasing the diffuser angle from 15 to 5 degrees in the numerical simulations, the critical pressure ratio increases and the discharge coefficient remains constant. By increasing the throat length of CVs, the critical pressure ratio decrease while discharge coefficient does not shown any changes.

Also, a variable area cavitating venturi was designed and investigated experimentally by Tian et al [11]. Four sets of experiments were conducted to investigate the effect of the pintle stroke, the upstream pressure and downstream pressure as well as the dynamic motion of the pintle on the performance of the variable area cavitating venturi. The obtained results verify that the mass flow rate is independent of the downstream pressure when the downstream pressure ration is less about 0.8. The mass flow rate is linearly dependent on the pintle stroke and increases with the upstream pressure. The discharge coefficient is a function of the pintle stroke; however it is independent of the upstream pressure. They concluded that the variable area cavitating venturi can control and measure the mass flow rate dynamically.

Zamoum and Kessal [12] have numerically investigate the dynamical of a bubbly flows in a transversal varying section duct (Venturi). The mass and momentum phases equations, which are coupled with the Rayleigh-Plesset equation of the bubbles dynamics are used. The effects of the throat dimension and the upstream void fraction on flow parameters are investigated. The numerical resolution of the previous equations set let us found that the characteristics of the flow change dramatically with upstream void fraction. Two different flow regimes are obtained: a quasi-steady and quasi-unsteady regime. The former is characterized by large spatial fluctuations downstream of the throat, which are induced by the pulsations of the cavitation bubbles. The quasi-unsteady regime corresponds to flashing flow in which occurs a bifurcation at the flow transition between these regimes.

The present work considers effects of the bubbles number and their distance on the oscillations parameters thought a Venturi nozzle. A non linear continuum bubbly mixture model coupled with the dynamic equation of N bubbles is used.

2 Basic Equation

The liquid is assumed to be incompressible and the interaction liquid duct wall is neglected. The total upstream bubbles population is uniform without coalescence, and the relative motion between the phases ignored. Gas and vapour densities are neglected in comparison to one of the liquid. The bubbles are assumed to have the same initial radius R_0 and external friction is neglected.

Then the mixture density can be expressed in function of bubble populationη:

$$\rho = \rho_L(1-\eta V)$$

Where $V = 4/3\pi R^3(x,t)$ is the volume of the bubble.

The dynamics of the bubbles can be modeled by the Rayleigh-Plesset equation

$$\rho_L^*\left[R^*\frac{d^2R}{dt^{*2}}+\frac{3}{2}\left(\frac{dR^*}{dt^*}\right)^2\right]+\frac{4\mu_E^*}{R^*}\frac{dR^*}{dt^*}=\left(p_s^*-p_v^*+\frac{2S^*}{R_s^*}\right)\left(\frac{R_s^*}{R^*}\right)^{3k}-p^*+p_v^*-\frac{2S^*}{R^*}-p_{ext}^* \quad (1)$$

Where $R^*(t)$ is the instantaneous bubble radius, R_s^* the upstream bubble radius, μ_E^* the effective dynamic viscosity of the liquid, ρ_L^* the density of the liquid, p_s^* the upstream pressure, p_v^* the vapor pressure, S^* the surface tension of the liquid, p^* the fluid pressure and p_{ext}^* is the imposed external pressure field, where:

$$p_{ext}^*=p_{si}^*+p_{A,i}(t) \quad (2)$$

Where

$$p_{si}^* = \sum_{j\neq 1}^{Nbub}\frac{\rho_L^*}{s_{ij}}\frac{d}{dt^*}\left(R_j^{*2}\frac{dR_j^*}{dt^*}\right)$$ is the pressure scattered by the other bubbles.

Where $s_{ij}=s_{ji}$ is the distance of bubble i from bubble j, $p_{A,i}(t)$ the applied pressure of any external field on bubble i.

Combining equations (1) and (2) gives the following coupled governing equation for coupled bubble oscillations,

$$\rho_L^*\left[R^*\frac{d^2R}{dt^{*2}}+\frac{3}{2}\left(\frac{dR^*}{dt^*}\right)^2\right]+\frac{4\mu_E^*}{R^*}\frac{dR^*}{dt^*}=\left(p_s^*-p_v^*+\frac{2S^*}{R_s^*}\right)\left(\frac{R_s^*}{R^*}\right)^{3k}-p^*+p_v^*-\frac{2S^*}{R^*}$$
$$-\sum_{j\neq 1}^{Nbub}\frac{\rho_L^*}{s_{ij}}\frac{d}{dt^*}\left(R_j^{*2}\frac{dR_j^*}{dt^*}\right)-P_{A,i}^*\left(t^*\right) \quad (3)$$

We assume that $s_{ij}=D=$ constant. Thus the distance of any bubble to any other bubble in the bubble population is constant. We further assume that the same external driving pressure field acts on all the bubbles, that is, $P_{A,1}(t^*)= P_{A,2}(t^*)= P_{A,3}(t^*)= P_{A,4}(t^*)= P_A(t^*)$, *then* $R_i^*(t^*)= R_j^*(t^*)= R^*(t^*)$.

Substituting into equation (3) yields:

$$\rho_L^*\left[R^*\frac{d^2R}{dt^{*2}}+\frac{3}{2}\left(\frac{dR^*}{dt^*}\right)^2\right]+\frac{4\mu_E^*}{R^*}\frac{dR^*}{dt^*}=\left(p_s^*-p_v^*+\frac{2S^*}{R_s^*}\right)\left(\frac{R_s^*}{R^*}\right)^{3k}-p^*+p_v^*-\frac{2S^*}{R^*}$$
$$-\left(N_{bub}-1\right)\frac{\rho_L^*}{D}\left(R^{*2}\frac{d^2R^*}{dt^{*2}}\right)-\left(N_{bub}-1\right)\frac{2\rho_L^*}{D}\left(R^*\left(\frac{dR^*}{dt^*}\right)^2\right)-P_A^*\left(t^*\right) \quad (4)$$

Equation (4) represents the idealized case where all the bubbles are equally spaced, have the same initial conditions.

The non-dimensional form equation (4) is giving by:

$$R\frac{D^2R}{Dt^2}+\frac{3}{2}\left(\frac{DR}{Dt}\right)^2+\frac{\sigma}{2}\left(1-R^{-3k}\right)+\frac{4}{Re}\frac{1}{R}\frac{DR}{Dt}+\frac{2}{We}\left(R^{-1}-R^{-3k}\right)+\frac{1}{2}Cp$$
$$+\left(N_{bub}-1\right)\frac{R_s^*u_s^{*2}}{D}R^2\frac{D^2R}{Dt^2}+\left(N_{bub}-1\right)\frac{2R_s^*u_s^{*2}}{D}R\left(\frac{DR}{Dt}\right)^2+P_A=0 \quad (5)$$

Where $D/Dt = \partial/\partial t + u\partial/\partial x$ is the Lagrangian derivative, $\sigma=\left(p_s^*-p_v^*\right)/1/2\rho_L^*u_s^2$ is the cavitation number, p_v^* is the partial pressure of vapor inside the bubble. $Re=\rho_L^*u_s^*R_s^*/\mu_E^*$ is the

Reynolds number, μ_E^* is the effective viscosity of liquid. $We = \rho_L^* u_s^{*2} R_s^* / S^*$ is the Weber number, S* is the liquid surface tension and ρ_L^* is the liquid density.
Continuity and momentum equations of the bubbly flow (Wang and Brennen 1998) [1] are:

$$\frac{\partial}{\partial t}\left[(1-\alpha)A\right]+\frac{\partial}{\partial x}\left[(1-\alpha)uA\right]=0 \qquad (6)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{1}{2(1-\alpha)}\frac{\partial Cp}{\partial x} \qquad (7)$$

Where $\alpha(x,t)=4/3\pi\eta R^3/\left[1+4/3\pi\eta R^3\right]$ is the bubble void fraction, *u(x, t)* the fluid velocity. $Cp(x,t)=\left(p^*(x,t)-p_s^*\right)/1/2\,\rho_L^* u_s^{*2}$ the fluid pressure coefficient, *p(x, t)* the fluid pressure, p_s^* the upstream fluid pressure, and u_s^* is the upstream fluid velocity.

Equations (5), (6) and (7) constitutes a simple model of one-dimensional flowing bubbles fluid with nonlinear bubbles dynamics.

2.1. Steady-State Solutions

Assuming steady-state conditions, all the partial time derivative terms in equations (5),(6) and (7) disappear. Then, the former equation set is transformed into an ordinary differential equation set, with only one independent variable (x):

$$(1-\alpha)uA=(1-\alpha_s)=constant \qquad (8)$$

$$u\frac{du}{dx}=-\frac{1}{2(1-\alpha)}\frac{dCp}{dx} \qquad (9)$$

$$R\left(u^2\frac{d^2R}{dx^2}+u\frac{du}{dx}\frac{dR}{dx}\right)\times\left(1+\frac{(N-1)}{D}R_s u_s^2 R\right)+\frac{3u^2}{2}\left(\frac{dR}{dx}\right)^2+2\frac{(N-1)}{D}R_s u_s^2 R u^2\left(\frac{dR}{dx}\right)^2+$$
$$\frac{4}{Re}\frac{u}{R}\frac{dR}{dx}+\frac{2}{We}\left(\frac{1}{R}-\frac{1}{R^{3k}}\right)+\frac{\sigma}{2}\left(1-\frac{1}{R^{3k}}\right)+\frac{1}{2}Cp=0 \qquad (10)$$

The corresponding initial conditions are:

R(x=0)=1, *U(x=0*)=1, *Cp(x=0*)=0

And the axial variation of the cross sectional takes the following from:

$$A(x)=\begin{cases}1 & 0<x<x_1\\ 1-\dfrac{(x-x_1)(1-\beta)}{x_2-x_1} & x_1<x<x_2\\ \beta & x_2<x<x_3\\ 1-\dfrac{(x_4-x)(1-\beta)}{x_4-x_3} & x_3<x<x_4\\ 1 & x_4<x<x_5\end{cases} \qquad (12)$$

Where β is the dimensionless radius of the Venturi throat and x the distance along the axis. In the present work it is assumed: $\beta=0.5$, $x_1=3.0$, $x_2=5.7$, $x_3=6.7$, $x_4=10.5$. This corresponds to an ISO standard Venturi (British Standards Institution) with a 21° converging section and 15° diverging section.

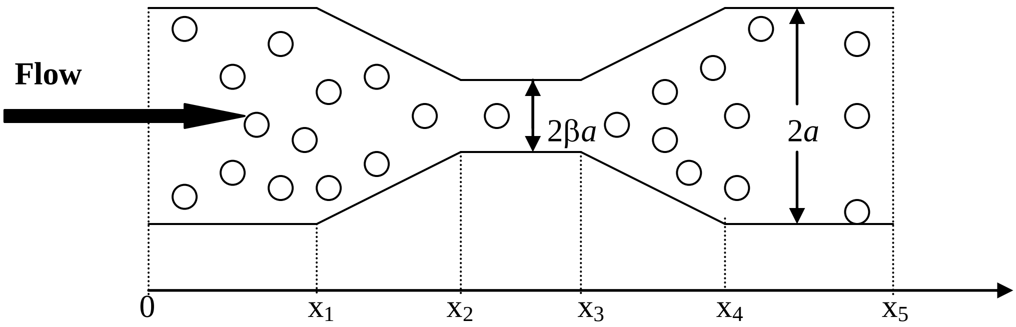

Figure 1: Cavitating flow through a Venturi.

3 Results and Discussion

Equation set (8)-(9) and (10) is resolved by the use of a fourth order Runge-Kutta scheme, with some flow conditions (Table 1).

Initial parameters	Water characteristics at 20°C
R_s^* =100µm u_s^* =10m/s k=1.4 Re=33 σ=0.8 We=137	ρ_L^* =1000Kglm^3 μ_E^* =0.03Ns/m^2 μ_L^* =0.001Ns/m^2 S*=0.073N/m

Table1. Initial condition flow and water characteristics

Effects of bubbles number is showed in figures (2). It can be observed here that the increasing of the bubbles number leads to frequency and the damping diminish of the signal.

Fluid axial velocity and pressure distribution are drawn in figure (3) and (4), for different bubbles number. It seems that the evolution of these parameters corresponds to the monophasic case.

The figure 5 shows the effect of distance between bubbles on the bubbles oscillations. The increase distance between bubbles has an important effect on the damping radius and frequency oscillations. However; for very large distances between bubbles, the radius distribution is similar for those of one bubbles evolution.

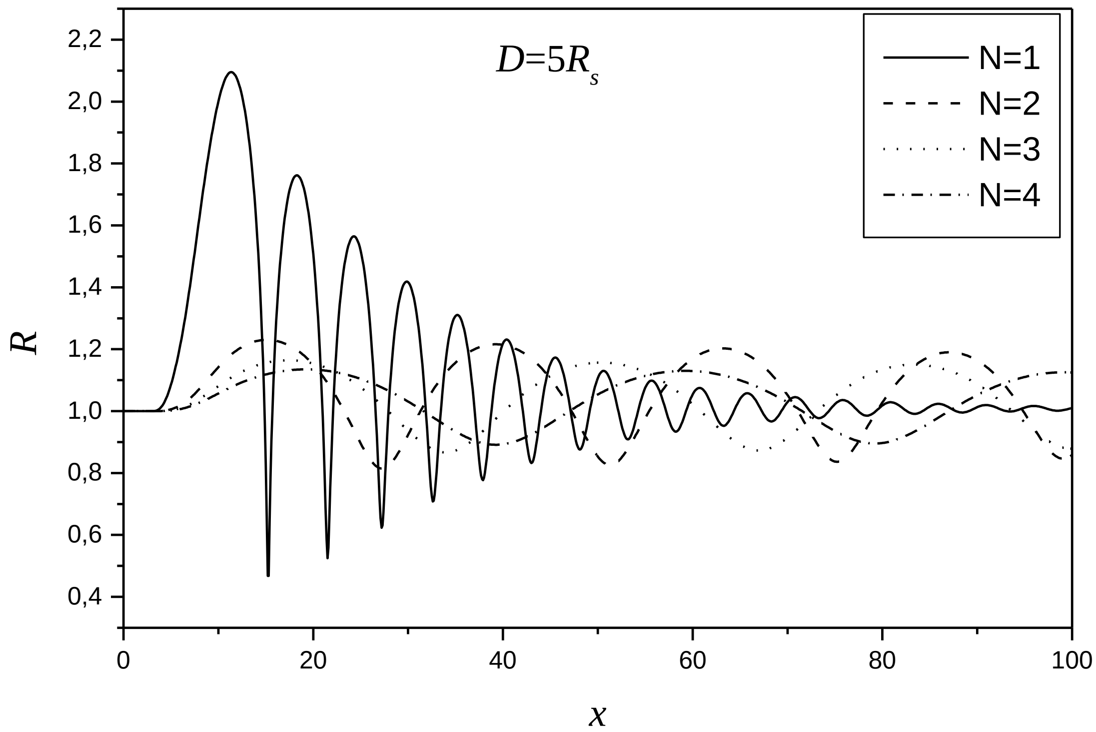

Figure 2: Bubbles radius for various values of upstream bubbles number

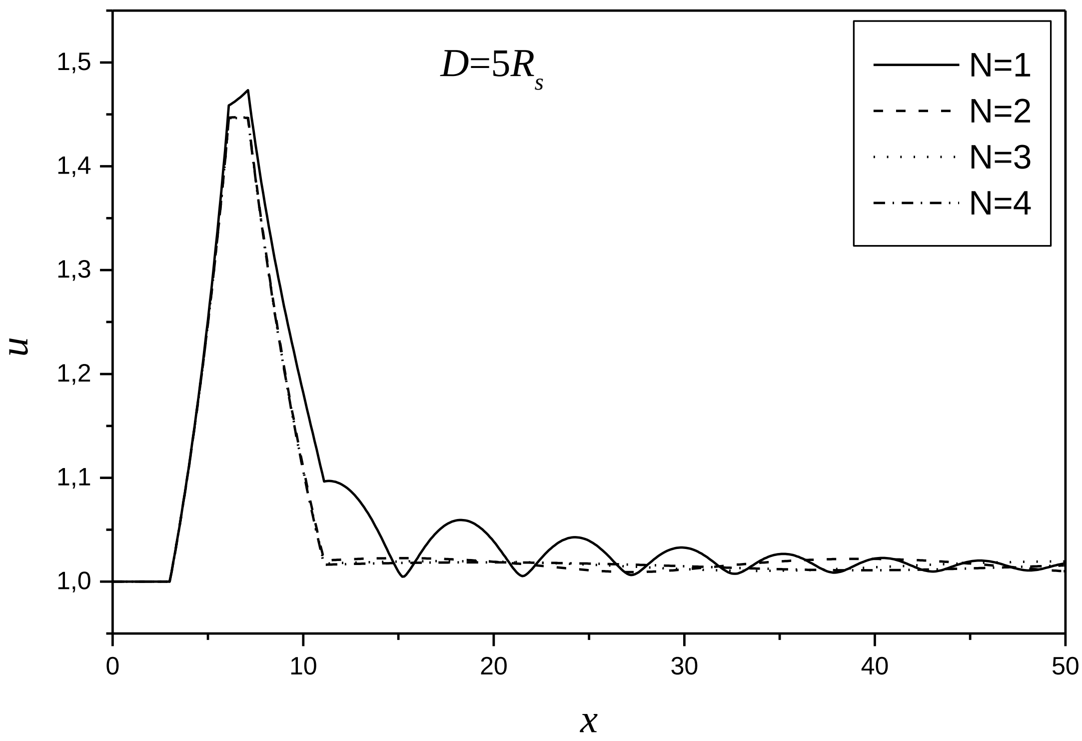

Figure 3: Fluid velocity for various values of the upstream bubbles number

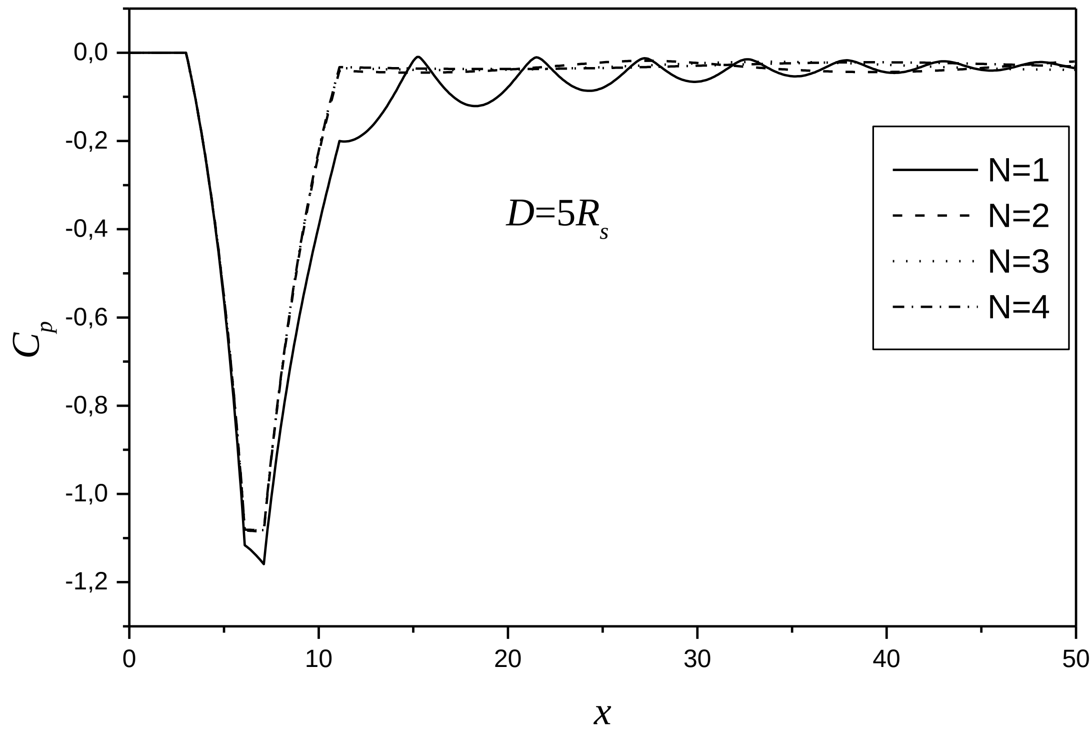

Figure 4: Fluid pressure for various values of the upstream bubbles number

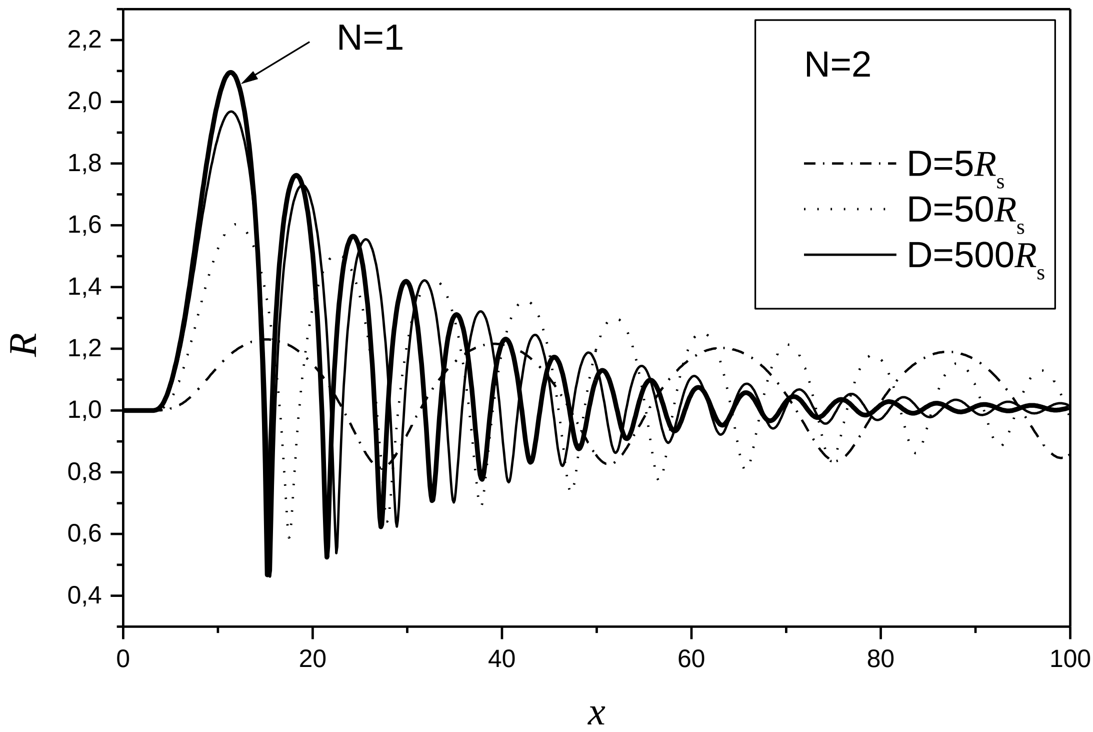

Figure 5: Bubbles radius for various values distance
of any bubble to any other bubble.

4 Conclusion

The new modele mass and momentum equation coupled with the Rayleigh-Plesset equation for a system of equally sised bubbles at the same distance from each other was derived, and is validated for the case small size distance between bubbles. Result found that the natural frequency and the damping of the bubbles system decrease as the number of bubbles increase. Also. The increase distance between bubbles has an important effect on the damping radius and frequency oscillations. However; for very large distances between bubbles, the radius distribution is similar for those of one bubbles evolution.

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