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# Numerical analysis of Fully Developed Laminar Flow and heat transfer of Non-Newtonian Fluid in Ducts of Arbitrary Cross-Sectional Shape

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**Abstract:** A numerical solution is presented for pressure drop and heat transfer characteristics of laminar fully-developed laminar flow of circular, square and triangular cross sections with subjected to a constant heat flux boundary condition. The numerical model is based on a 3D Navier–Stokes incompressible flow and energy equation. Results for the velocity field, Poiseuille number and Nusselt number product are computed for different power-law indices ($0.5 \leq n \leq 1.0$), which are tabulated and graphically presented. For each ducts studied, one induces higher heat transfer intensification with a strong pressure loss. With an alternative duct, a better compromise between heat transfer and pressure loss is obtained. Critical comparisons with previous results in the literature are also performed, in order to validate the numerical codes developed in the present work and to demonstrate the consistency of the final results.

*Keywords:* Laminar flow, non-Newtonian fluids, Poiseulle number and Nusslt number.

## 1 Introduction

Fully developed laminar flows inside straight ducts havebeen a subject of great interest as demonstrated by the currently literature, mainly to the interest in flows within concentric circular ducts. This problem was initially considered by Balmer and Fiorina [1].

A brief literature survey indicates that Lin and Shah [2] (1978) have studied the laminar flow problem of power-law fluids with yield stress, flowing in the entrance region of a circular duct and of parallel plates; they used a forward marching procedure to solve the related momentum. Z. Warsi (1994) [3] investigated the laminar flow of power-law fluids through circular pipes, this study concerned with the flow of power-law fluids through circular pipes under impulsively started conditions both for constant and superposed fluctuating pressure gradients. Cuccurullo and Berardi [4] (1998) investigated the simultaneously developing of velocity and temperature profiles in the entrance pipe flow. The flow was assumed to be steady state for a non-Newtonian fluid in incompressible laminar pipe flow. The fluid behavior was assumed to follow the Ostwald-de Waele power-law model. The developing velocity and temperature profiles were solved by the integral method. Results were presented and discussed in terms of axial and radial velocity profiles, Fanning friction factors and Nusselt numbers for different fluid properties and thermal boundary conditions.

R. A. Brewster et al (1987) [5] has studied numerically laminar flow of modified power law fluids. Results presented the Poiseuille number relation for a rheological fluid which has Newtonian behavior at low shear rates, power law behavior at high shear rates and a transition zone in between.

Computational studies of Newtonian and non-Newtonian have been carried out on pipe flows subject to a general time-dependent pressure gradient by S. McGinty et al (2009) [6]. The study concerned with general analytic solutions of flows in cylindrical and annular pipes with an arbitrary time-dependent pressure gradient and arbitrary steady initial flow. The fluids considered are Newtonian, Maxwellian and Oldroyd B. Graphical results for (blood) flow in a dog's femoral artery are presented

Syrjala has studied (1995) [7] the fully developed laminar flow of power-law non-Newtonian flow in a rectangular straight duct using the finite element method. Laminar heat transfer in the entrance region of a circular duct and parallel plates has been studied by Nguyen (1992) [8] by ADI and QUICK methods.

Shah (1975) [9] have investigated the Laminar flow forced convection for Newtonian flow by using analytical method inside curved ducts. Montgomery and Wilbulswas (1966) [10] solved the thermal flow problem for rectangular ducts by using the explicit finite difference method. Entrance region heat transfer in rhombic ducts has been studied by Asako and Faghri (1988) [11] for a Newtonian fluid by algebraic coordinate transformation. Hydrodynamically developed channel flow and heat transfer to power-law fluids have been studied by Ashok and Sastri (1977) [12] for a square duct under three thermal boundary conditions.

In this context, the present study applies the development of Newtonian and non-Newtonian laminar flow for Power law fluids flowing in the entrance region of circular, square and triangular straight ducts, and for this purpose, the index flow varying from 0.5 to 1.0.

# 2 Physical model

Laminar flow inside three-dimensional circular, square and triangular ducts. The figure below shows the mesh used for all the cases have quadrilateral elements with a very fine grid spacing. The mesh shown in figure 1 has about 260000 elements.

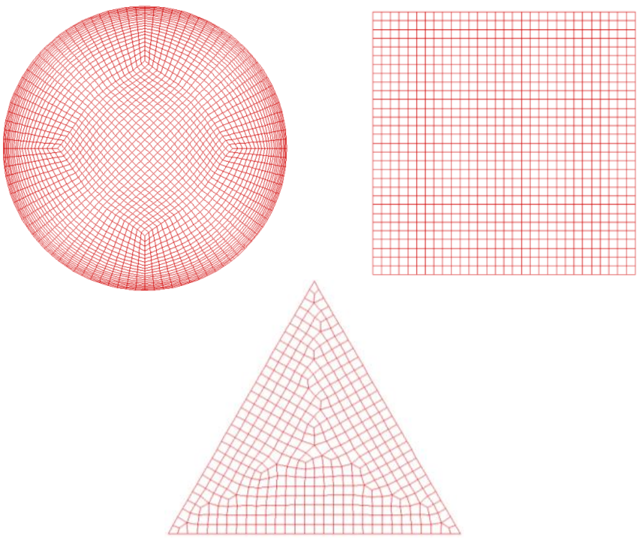

Figure: 1Grid of different model geometries cross-sections

# 3 Analysis

In order to model the flow of non-Newtonian fluids, the purely viscous (i.e., inelastic) non-Newtonian character of the fluid that is studied here is represented by Bird et al [13] power-law model for the case of both shear-thinning and Newtonian fluids. The constitutive relation between the shear stress $\tau$ (Pa) and the shear rate $\gamma$ (s−1) can be described by a simple power law expression:

$$\tau = k\gamma^{n} \qquad (1)$$

Where k (Pa-1) is power-law consistency index and n is the flow behavior index of the fluid.

The nonlinear relationship between the apparent viscosity $\mu_{app}$ (Ns $m^{-2}$) and the shear rate $\gamma$ is given by the constitutive equation:

$$\mu_{app} = k\gamma^{n-1} \qquad (2)$$

We will consider three different flow behavior indexes that are associated with shear-thinning (n = 0.5 and 0.8) and Newtonian (n = 1) fluids. The consistency index (k) is adapted in each non-Newtonian case in order to give the same generalized Reynolds number as was considered for the Newtonian flow ( $\mathrm{Re} = \frac{\rho U_i D_h}{\mu}$ ). This generalized Reynolds number ($Re_g$), can be written for the power-law fluid as:

$$\mathrm{Re}\, g = \frac{\rho U_i^{2-n} D_h^n}{k} \qquad (3)$$

Where, $\rho$ (Kg/m3) is density of fluid (kg m−3), n is the power-law index, and Ui (m/s) is the inlet velocity.

Poiseuille number (Po) :

The hydrodynamic performance of all geometries is characterized by the evolution along the curvilinear coordinate s of the local friction coefficient f, defined as:

$$f = \frac{2\left(\frac{dp}{ds}\right) D^{h}}{\left(\rho U_i^2\right)} \qquad (4)$$

Where, dp/ds is the local pressure gradient along the curvilinear coordinate of the channel. Because this parameter depends on the Reynolds number, it is preferable to follow the evolution of the Poiseuille number,

$$Po = f.\mathrm{Re}_g \qquad (5)$$

Nusselt number:

The Nusselt number is defined as:

$$Nu = \frac{q_w}{\left(T_b - T_w\right)} \frac{d_h}{\lambda} \qquad (6)$$

Where, qw (w/m2) is the wall heat flux, Tb (k) is the mean bulk temperature fluid over the cross-sectional area, Tw (k) is perimeter average wall temperature and $\lambda$ (w/mk) is the thermal conductivity.

# 4 Numerical Solution Methodology

The conservation equations for mass, momentum and energy were solved by using a computational fluid dynamics (CFD). The standard scheme is used for pressure discretization, and the SIMPLE scheme is employed for pressure-velocity coupling. The momentum and energy equations are solved with second-order up-wind scheme. The computations were considered to be converged once all the scaled residuals are less than $10^{-7}$ and the global imbalances, representing overall conservation, don’t exceed $10^{-5}$.

# 5 Results and discussion

In the present work, results were obtained for three-dimensional fully developed laminar flow of Power law model with different values of power-law indices, namely n = 0.5; 0.6; 0.7; 0.8; 0.9 and1.0, for three different configurations (circular, square and triangular ducts).

## 5.1 Validation

In this section, in order to check the reliability and the precision of the CFD computation, a comparison with other results provided in the literature is carried out. A fully developing laminar steady flow of non-Newtonian power-law fluid in straight channel with square cross section is considered.

Tables 1 and 2 show, respectively, a comparison of the values of the Poiseuille number and the Nusselt number obtained in the present work and those provided in the literature for different power-law index (n=0.5-1). The numerical values barely differ from the case of the theoretical values where the maximum difference is less than 0.5%. These values are in fair agreement and the comparison is satisfactory and reveals a very good concordance.

Table. 1 Poiseuille number, Po, of fully developed laminar flow in square straight channel for different power-law index (n=0.5-1).

Tab. 1 Poiseuille number, Po, of fully developed laminar flow in square straight channel for different power-law index (n=0.3-1).

| n | 1 | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 |
|---|---|---|---|---|---|---|
| Present work | 56.90 | 47.47 | 39.32 | 33.00 | 27.52 | 22.90 |
| Wheeler and Wissler [13] | 56.92 | 47.53 | 39.67 | 33.07 | 27.54 | 22.89 |
| Seppo [14] | 56.90 | 47.52 | 39.65 | 33.06 | 27.53 | 22.88 |
| Simsoo et al [15] | 56.90 | 47.89 | 40.29 | 33.89 | 28.49 | 23.91 |
| Kozicki et al [16] | 56.91 | 47.88 | 40.26 | 33.82 | 28.37 | 23.75 |
| Sayed-Ahmed [17] | 56.90 | - | - | - | - | 22.88 |
| Error (%)/ [14] | 0.007 | 0.09 | 0.82 | 0.17 | 0.05 | -0.06 |

Table. 2 Nusselt number of fully developed laminar flow in square straight channel for different power-law index (n = 0.5-1)

| n | 1 | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 |
|---|---|---|---|---|---|---|
| Present work | 3.07 | 3.114 | 3.14 | 3.18 | 3.22 | 3.28 |
| Wheeler and Wissler [13] | 3.09 | 3.10 | 3.13 | 3.17 | 3.21 | 3.27 |
| Error (%)/[13] | 0.24 | -0.25 | -0.36 | -0.38 | -0.37 | -0.238 |

### 5.2 Flow visualization

To discuss the whole velocity and temperature fields, figure 2 shows contour maps of the magnetic velocity distributions and static temperature of outflow section at different power-law index n = 1 and n = 0,5 for all cases geometry.

Figure 3 illustrates the development of the magnitude velocity profiles for different power-law indices, namely, n = 0.5; 0.6; 1.7; 0.8; 0,9 and 1.0, as function of the X-coordinate.

From this figure it can be noticed that when the power-law index increases there is an increase in the value of the center line velocity for all models. In regions near the tube wall, it is verified that the velocity gradient diminishes as n increases. This is due to an increase of the apparent fluid viscosity, and consequently an increase of the wall stress. For practical engineering considerations, this effect leads to an undesirable increase of the pumping power to promote the flow of this type of fluids inside triangular ducts.

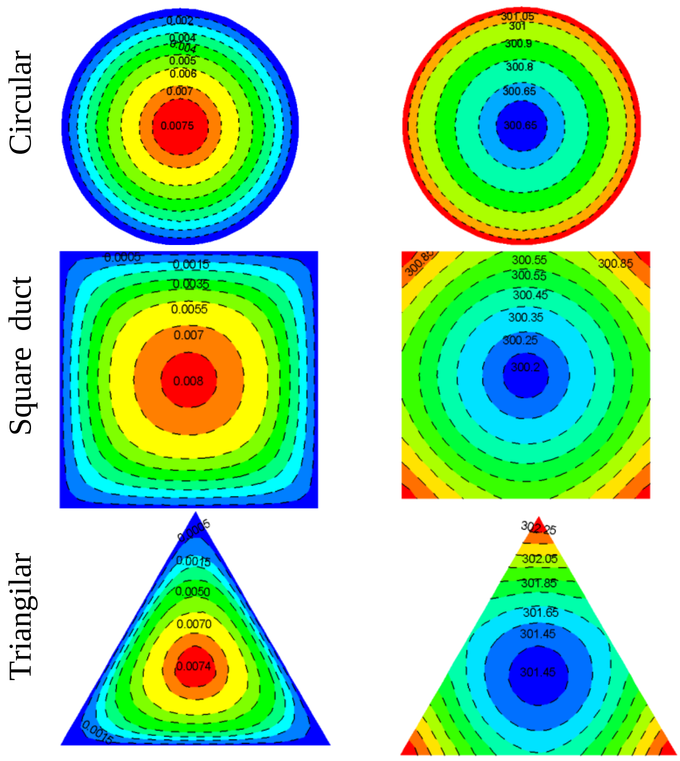


Figure: 2 Contours of magnitude-velocity (left) and static temperature (right) of different geometries at n = 0.5

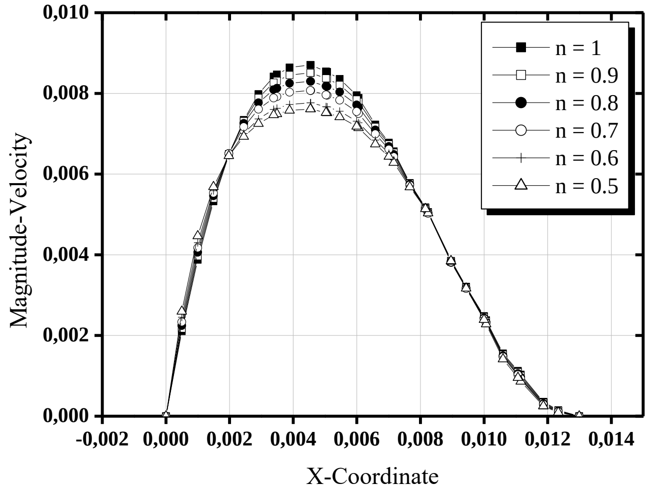


Figure:3 Centerplane velocity profiles for non-Newtonian fluids in a triangular duct.

Table 3 shows the values of Poiseuille number and average Nusselt number for different cross sectional ducts with various power-law index (n = 1-0.5)

Poiseuille numbers (Po) were determined numerically (as shown in table 3) with the measured pressure differences and average velocities through the circular, square and triangular ducts for different flow index. For power-law fluids (n = 0.5), the value of the Poiseuille numbers decrease more than the Newtonian fluid (n = 1).

Tab. 3 Poiseuille number and Nusselt number of fully developed laminar flow in circular, square and triangular straight channel for different power-law index (n = 0.5-1)

| | Po | | Nu | | Nu/Po | |
|---|---|---|---|---|---|---|
| n | 1 | 0.5 | 1 | 0.5 | 1 | 0.5 |
| Circular duct | 63.99 | 25.19 | 1.88 | 1.97 | 0.02 | 0.07 |
| Square duct | 56.76 | 28.96 | 3.07 | 3.27 | 0.05 | 0.11 |
| Triangular duct | 53.28 | 21.92 | 4.36 | 4.37 | 0.081 | 0.19 |

# 7 Conclusion

The hydrodynamically fully developed and thermally developing laminar flow of Newtonian and non-Newtonian power-law fluid in arbitrary cross-sectional ducts was studied in this work. For each ducts studied, one induces higher heat transfer intensification with a strong pressure loss. With an alternative duct, a better compromise between heat transfer and pressure loss is obtained.
The numerical results shows that a non-Newtonian fluid with a flow behavior index of less than one gives a higher heat transfer coefficient than a Newtonian fluid. For example, the Nusselt number was found to be 3.274 for n=0.5 and 3.070 for n=1 in square duct. Due to the reduction in friction power requirement and the increase in heat transfer rates, Power-law fluids seem to be better working fluids in heat exchange equipment than Newtonian fluids. Despite this, the ratio of the Poiseuille number to the Nusselt number is higher in the triangular duct, showing that the heat transfer enhancement is important than the pressure loss increase.

# Nomenclature

D hydraulic diameter, m
K power-law consistency index, $(Pa.s^{-1})$
n power-law index
Po Poiseuille number
Re Reynolds number for Newtonian fluid
$Re_g$ Reynolds number for non-Newtonian fluid
T fluid temperature, K
Ui mean velocity at the inlet section, $m.s^{-1}$
x,y,z vertical coordinates in Cartesian coordinate system, m

Grec Symbol

$\dot{\gamma}$ shear rate, $(s^{-1})$
$\tau$ shear stress, (Pa)
$\mu$ constant viscosity for Newtonian fluid, $(N\ s\ m^{-2})$
$\mu_{app}$ apparent viscosity for non-Newtonian fluid, $(N\ s\ m^{-2})$
$\rho$ fluid density $(kg\ m^{-3})$

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