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Numerical Solution of Burgers-Huxley Equation using Improved Nodal Integral Method

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Abstract: In this paper, a numerical scheme for solution of Generalized Burgers-Huxley equation using improved Nodal integral method (MNIM) is proposed. In this approach, the domain is divided into nodes and the derivation of the scheme is based on local transverse integration process (TIP) of PDEs within these nodes. A set of algebraic equations, based on the local analytical solution of ODEs obtained by TIP, is derived. The results are obtained by developed method for several values of the parameters. These results are compared with analytical solution and the results of few other well established methods. It is observed that the current method is capable of capturing the sharp wave front with quite coarse grids more accurately in comparison to the traditional numerical approaches.

Keywords: Improved Nodal Integral Method, coarse node method, shock wave, Burgers-Huxley Equation, and Nonlinear equation.

1 Introduction

Nonlinear partial differential equations are often encountered in major areas such as physics, chemistry, mathematics and engineering. The majority of the real life problems are nonlinear whose solutions are quite challenging. Researchers have developed many schemes, however, there is still significant scope for improvement in computational efficiency. For improving computational capacity, there is a need to develop new schemes, improve existing numerical methods and develop efficient solvers. Nodal Integral method (NIM) is a preferred accurate coarse grid method. NIM was used to solve neutron transport equations in early 1970s [1] and it is found that the scheme is able to produce accurate results with relatively coarser grids as compared to traditional methods. The key step involved in NIM is the transverse integration process (TIP) [2] which reduces the PDEs into sets of approximate ODEs that can be solved analytically within a node. The final sets of difference equations depend on the analytical solution of these ODEs. This local approximate analytical solution is one of the main distinguishing features of NIM which significantly increases the accuracy of the scheme. The TIP process leads to more number of discrete unknowns in each cell or node for NIM but the overall number of discrete unknowns in the entire geometry are less due to the use of coarser grid which nullify the effect of more unknowns per node. In other words, for a given node size, a second-order coarse node scheme gives more accurate solutions than traditional methods. However, these schemes depend on the transverse-integration which restricts the scheme to region with

boundaries parallel to axis, i.e. in rectangular geometries. The simplicity in developing procedure for NIM stresses its importance with possible extension of its applications in the fields of fluid flow, heat transfer and other branches of physics and engineering. To show the accuracy and efficiency of NIM, nonlinear PDE known as Generalized Burgers-Huxley equation (GBHE) is chosen as a test problem. GBHE is very commonly used to describe the interaction between reaction mechanisms, convection effects and diffusion transport [3] [4], nerve propagation [5], wall motion in liquid crystals, population dynamics [6], chemical kinetics, electrodynamics, transport phenomena [7] . Burger's equation is one of the reduced forms of Burgers –Huxley equation which is used as a model equation to describe shock wave formulation, sound wave and traffic flow [8]. Burgers equation has often been used as a simplified form of the Navier-Stokes equation (without pressure terms) [5] and has been used widely for comparison of numerical schemes in different studies. A similar approach has been used for solving Burger Huxley equations [9]. However, in the current method some improvements have been made to the above mentioned method. Moreover, a comparison with the existing schemes have been carried out in the present work. In the present work, the solutions of the NIM scheme developed for GBHE are compared with analytical solution as well as those obtained with other numerical schemes. The comparison is carried out to demonstrate the capability of NIM to give more accurate solutions with coarse grids.

2 Problem Statement

The generalized Burgers –Huxley equation [10]-[11] is given as,

$$\frac{\partial U}{\partial t}+\alpha U^{\delta}\frac{\partial U}{\partial x}-\frac{\partial^2 U}{\partial x^2}=\beta U(1-U^{\delta})(U^{\delta}-\gamma) \qquad -l\le x\le +l, \qquad t\ge 0 \tag{1}$$

and exact solution is given in [11,12] as,

$$U(x,t)=\left(\frac{\gamma}{2}\left\{1-tanh\left(a_1(x-a_2t)\right)\right\}\right)^{\frac{1}{\delta}} \tag{2}$$

where a1 and a2 are given by,

$$a_1=\frac{-\alpha\delta\pm\delta\sqrt{\alpha^2+4\beta(1+\delta)}}{4(1+\delta)} \qquad a_2=\frac{\alpha}{1+\delta}-\frac{(1-\gamma-\delta\gamma)(\alpha\pm\sqrt{\alpha^2+4\beta(1+\delta)})}{2(1+\delta)}$$

Here, U is the velocity and *α, β, γ, δ* are the parameters, $\beta\ge 0,\delta>0,\gamma\in(0,1)$. When $\alpha=0,\delta=1$ it reduces to Huxley equation and when $\beta=0$, equation reduces to Generalized Burgers Equation [11]

3 Formulation: Modified Nodal Integral method

One dimensional generalized Burgers'-Huxley equation is given by**,**

$$\frac{\partial U}{\partial t}+\alpha U^{\delta}\frac{\partial U}{\partial x}-\frac{\partial^2 U}{\partial x^2}=\beta U(1-U^{\delta})(U^{\delta}-\gamma) \qquad \forall -l\le x\le +l \ \ \&\ \ t\ge 0 \tag{3}$$

where U is velocity in x-direction, α, β, γ, δ are different parameters. The entire domain is divided into 'n' nodes over (x, t) plane as shown in Figure 1 of size '$2a_{i,j}$' in space and '$2\tau_{i,j}$' in time, respectively. Each of these nodes have origin at its center. Each of these nodes have origin at its

center. Now Eq. (3) is averaged over time by operating with $\frac{1}{2\tau}\int_{-\tau}^{+\tau} dt$, and averaged over space by operating with $\frac{1}{2a}\int_{-a}^{+a} dx$ for each node, respectively. This averaging of equation with in a node is known as TIP.

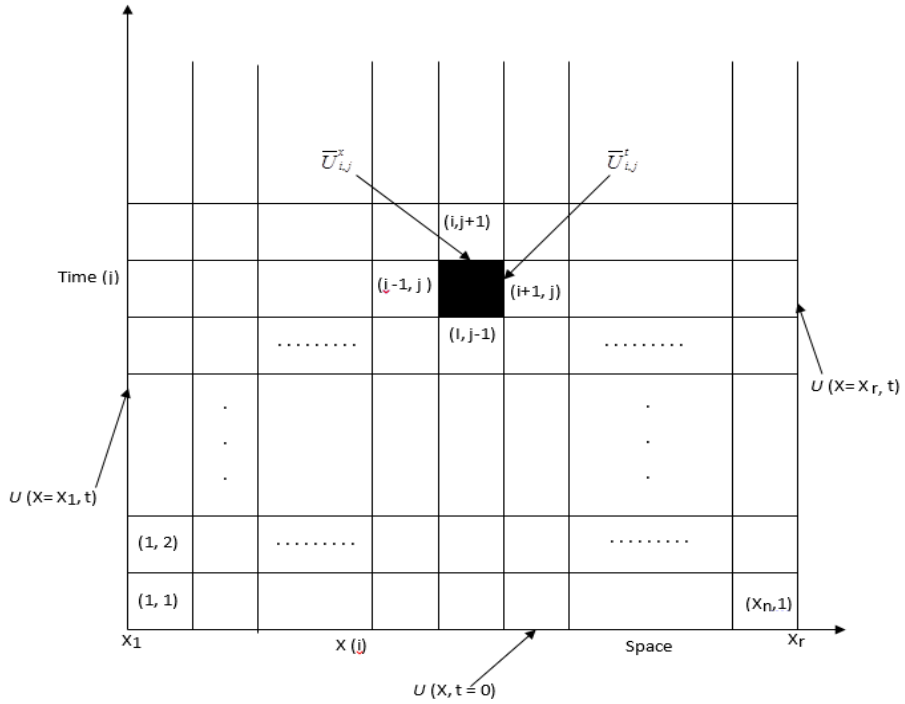


Figure 1. Coordinate system and the relation between velocities with node

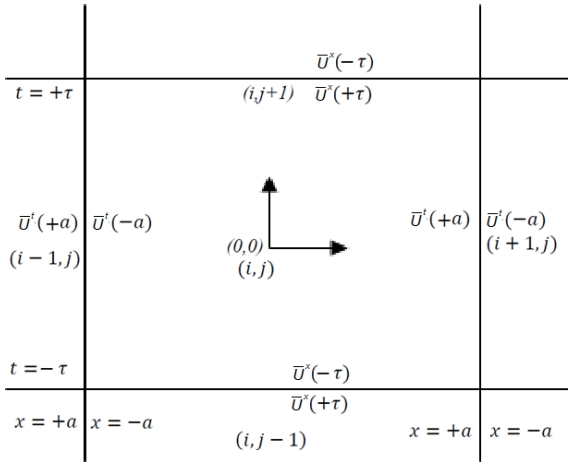


Figure 2. Coordinate system with in operational node.

TIP gives number of ODEs (per node) equal to the number of independent variables.

$$\frac{1}{2\tau}\int_{-\tau}^{+\tau}\left[\frac{\partial U}{\partial t}+\alpha U^{\delta}\frac{\partial U}{\partial x}-\frac{\partial^2 U}{\partial x^2}\right]\mathrm{dt}=\frac{1}{2\tau}\int_{-\tau}^{+\tau}\beta U\left(1-U^{\delta}\right)\left(U^{\delta}-\gamma\right)dt \qquad (4)$$

$$\frac{1}{2a}\int_{-a}^{+a}\left[\frac{\partial U}{\partial t}+\alpha U^{\delta}\frac{\partial U}{\partial x}-\frac{\partial^2 U}{\partial x^2}\right]\mathrm{dx}=\frac{1}{2a}\int_{-a}^{+a}\beta U\left(1-U^{\delta}\right)\left(U^{\delta}-\gamma\right)dx \qquad (5)$$

In this process, index (i,j) are dropped for the sake of simplicity in development and it is subsequently introduced whenever necessary. While performing TIP, average of product is approximated as product of averages for nonlinear term which will leads to a second order approximation. Now defining $\bar{U}^t(x) = \frac{1}{2\tau_{i,j}} \int_{-\tau_{i,j}}^{+\tau_{i,j}} U dt$ and using following approximation [1],

$$\left[\frac{1}{2\tau}\int_{-\tau}^{+\tau} U^{\delta} dt\right] \approx \left[\frac{1}{2\tau}\int_{-\tau}^{+\tau} U(x,t) dt\right]^{\delta} = \left(\bar{U}_{i,j}^{t}(x)\right)^{\delta} \approx \left(\left(\bar{U}_{i,j}^{t} + \bar{U}_{i-1,j}^{t}\right)/2\right)^{\delta} = U_{0(i,j)} \quad \textbf{(6)}$$

Nonlinear term in the equation is linearized by considering time averaged velocities $\bar{U}^t(x)$ to be constant over nodes. By using the approximation stated above, the averaging of Eq. (3) over time, yields,

$$\frac{d^2\bar{U}^t(x)}{dx^2} - \alpha U_0 \frac{d\bar{U}^t(x)}{dx} = \bar{S}_2^t(x), \quad \textbf{(7)}$$

Note that the remaining terms in eq. 3 are incorporated in pseudo source term $\bar{S}_2^t(x)$. Similarly for space averaged velocity $\bar{U}_{i,j}^x$,

$$\frac{d\bar{U}^x(t)}{dt} = \bar{S}_1^x(t) \quad \textbf{(8)}$$

Here, $\bar{S}_1^x(t)$ is the space averaged pseudo source term in the node (i,j) and $\bar{S}_2^t(x)$ is the time averaged pseudo source term in the node (i,j) which is a only function of time and space, respectively. The pseudo-source terms are now expanded in Legendre polynomials and truncated at zeroth order which leads to constant pseudo source terms. On integrating equation (7) and (8) and applying the nodal boundary conditions as shown in the Figure 2, one gets,

$$\bar{U}_{i,j}^x = \bar{U}_{i,j-1}^x + 2\tau \bar{S}_{1i,j}^{x,t} \quad \textbf{(9)}$$

$$\bar{U}_{i,j}^t = \bar{S}_{2i,j}^{x,t}\left[\frac{a}{U_0} - \frac{x}{U_0} + \frac{2ae^{\frac{U_0 x}{\alpha}}}{\alpha U_0\left(e^{\frac{U_0 a}{\alpha}} - e^{-\frac{U_0 a}{\alpha}}\right)}\right] + \left[\frac{\bar{U}_{i-1,j}^t e^{\frac{U_0 a}{\alpha}} - \bar{U}_{i,j}^t e^{-\frac{U_0 a}{\alpha}} + \bar{U}_{i,j}^t e^{\frac{U_0 x}{\alpha}} - \bar{U}_{i,j}^t e^{\frac{U_0 x}{\alpha}}}{\left(e^{\frac{U_0 a}{\alpha}} - e^{-\frac{U_0 a}{\alpha}}\right)}\right] \quad \textbf{(10)}$$

In the development, till now all nodes are independent of each other. For developing the relation between two consecutive nodes, continuity is applied at the interface of two nodes, i and $i+1$. Using continuity on the solution given in equation 10 and with the velocity notation as shown in Figure 3, one gets Eq. (11). The obtained equation is a three point scheme.

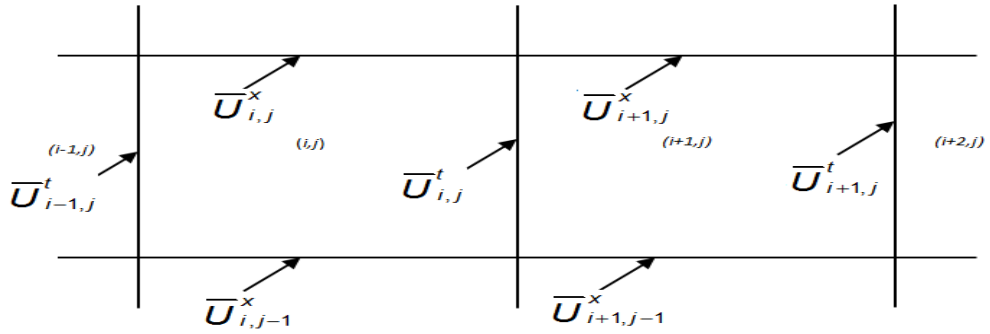


Figure 3. Averaged quantities showing interdependency with in adjacent nodes

$$\overline{U}^{t}_{i-1,j}\left[\frac{U_{0(i,j)}/\alpha}{\left\{1-e^{-\frac{2U_{0(i-1,j)}a}{\alpha}}\right\}}\right]-\overline{U}^{t}_{i,j}\left[\frac{\frac{U_{0(i,j)}}{\alpha}}{\left\{1-e^{-\frac{2U_{0(i,j)}a}{\alpha}}\right\}}+\frac{\frac{U_{0(i+1,j)}}{\alpha}}{\left\{e^{\frac{2U_{0(i+1,j)}a}{\alpha}}-1\right\}}\right]+\overline{U}^{t}_{i+1,j}\left[\frac{\frac{U_{0(i+1,j)}}{\alpha}}{\left\{e^{\frac{2U_{0(i+1,j)}a}{\alpha}}-1\right\}}\right]=$$

$$\frac{\overline{S}^{x,t}_{2i,j}}{\alpha}\left[\frac{2a}{\left\{1-e^{-\frac{2U_{0(i,j)}a}{\alpha}}\right\}}-\frac{\alpha}{U_{0(i,j)}}\right]+\frac{\overline{S}^{x,t}_{2i+1,j}}{\alpha}\left[\frac{\alpha}{U_{0(i+1,j)}}-\frac{2a}{\left\{e^{\frac{2U_{0(i+1,j)}a}{\alpha}}-1\right\}}\right] \quad \textbf{(11)}$$

From equation 9 and 11, there are two equations and four unknowns ($\overline{U}^{x}_{i,j},\overline{U}^{t}_{i,j},\overline{S}^{t,x}_{2i,j},\overline{S}^{x,t}_{1i,j}$) thus two extra constraints are needed to evaluate all the four unknowns. First constraint equation is obtained by averaging the original equation with respect to space and time. It gives the relation between pseudo source terms.

$$\frac{1}{4a\tau_{i,j}}\int_{-\tau-a}^{+\tau+a}\int\left\{\frac{\partial U}{\partial t}+\alpha U^{\delta}\frac{\partial U}{\partial x}-\frac{\partial^2 U}{\partial x^2}\right\}dx\,dt=\frac{1}{4a\tau_{i,j}}\int_{-\tau-a}^{+\tau+a}\int\left\{\beta U(1-U^{\delta})(U^{\delta}-\gamma)\right\}dx\,dt \quad \textbf{(12)}$$

The RHS of above equation 12 can be approximated by considering average of product equal to product of averages, where as in [9] different approximation has been proposed for RHS.

$$\frac{1}{4a\tau}\int_{-\tau-a}^{+\tau+a}\int\left\{\beta U\left(1-U^{\delta}\right)\left(U^{\delta}-\gamma\right)\right\}dtdx=\beta U_0\left(1-U_0^{\delta}\right)(U_0^{\delta}-\gamma)=\overline{F}^{x,t}_{i,j} \quad \textbf{(13)}$$

$$\overline{S}^{t,x}_{2i,j}=\overline{S}^{x,t}_{1i,j}+\overline{F}^{x,t}_{i,j} \quad \textbf{(14)}$$

and the second constraint is obtained by enforcing the uniqueness of the node averaged variable i.e.,

$$\overline{U}^{x}_{i,j},=\overline{U}^{t}_{i,j} \quad \textbf{(15)}$$

On solving equation (9) and (11) by applying the constraints given by equations (14) and (15), we get the final sets of difference equation per node

$$\left(\frac{C_{i,j}-\tau}{\tau}\right)\overline{U}^{x}_{i,j}-\left(\frac{C_{i,j}+\tau}{\tau}\right)\overline{U}^{x}_{i,j-1}+2A_{i,j}\overline{U}^{t}_{i,j}+2B_{i,j}\overline{U}^{t}_{i-1,j}=2\overline{F}^{x,t}_{i,j}C_{i,j} \quad \textbf{(16)}$$

$$\left(E_{i+1,j}\right)\overline{U}^{t}_{i+1,j}-\left(F_{i,j}+E_{i,j}\right)\overline{U}^{t}_{i,j}+\left(F_{i,j}\right)\overline{U}^{t}_{i-1,j}-\left(\frac{D_{i,j}}{2\tau}\right)\left(\overline{U}^{x}_{i,j}-\overline{U}^{x}_{i,j-1}\right)$$
$$-\left(\frac{2aA_{i,j}}{2\tau}\right)\left(\overline{U}^{x}_{i+1,j}-\overline{U}^{x}_{i+1,j-1}\right)=-2aA_{i+1,j}\overline{F}^{x,t}_{i+1,j}+\overline{F}^{x,t}_{i,j}D_{i,j} \quad \textbf{(17)}$$

where, $A_{i,j}=\left\{\frac{\alpha}{2U_{oi,j}a}-E_{i,j}\right\}$ $\qquad B_{i,j}=1-A_{i,j}$

$C_{i,j}=\frac{1}{\alpha}\left\{\left(\frac{\alpha}{U_{oi,j}}\right)^2-\frac{\alpha a}{U_{oi,j}}-\frac{2\alpha aE_{i,j}}{U_{oi,j}}\right\}$ $\qquad D_{i,j}=2aF_{i,j}-\frac{\alpha}{U_{oi,j}}$,

$E_{(i,j)}=\frac{1}{\left[e^{2\left(\frac{U_{oi,j}a}{\alpha}\right)}-1\right]}$ $\qquad F_{i,j}=\frac{1}{\left[1-e^{-2\left(\frac{U_{oi,j}a}{\alpha}\right)}\right]}$

4 Numerical Results

Modified Nodal integral method (MNIM) is used to develop the numerical scheme for solution Burgers-Huxley equation. MNIM is a coarse node method which is based on the analytical solution of ODEs within the node. Picard type method is used to solve the developed scheme. To show the effectiveness of the developed method comparison with analytical solution as well as various other developed schemes is done. In all the cases,

the initial condition used is, $U(x,0)=\left(\frac{\gamma}{2}\{1-tanh(a_1 x)\}\right)^{\frac{1}{\delta}}$, **(18)**

the Left boundary condition used is, $U(-l,t)=\left(\frac{\gamma}{2}\{1-tanh(a_1(-l-a_2 t)\}\right)^{\frac{1}{\delta}}$, **(19)**

and the right boundary condition used is, $U(+l,t)=\left(\frac{\gamma}{2}\{1-tanh(a_1(+l-a_2 t)\}\right)^{\frac{1}{\delta}}$. **(20)**

The parameters taken for comparison are α = 1, β = 1, γ = 1, δ = 1 and 2, dt = 0.001 and 0.01 with different grid sizes as well as different integral time steps. Here all simulations are done up to time, T = 6 units.

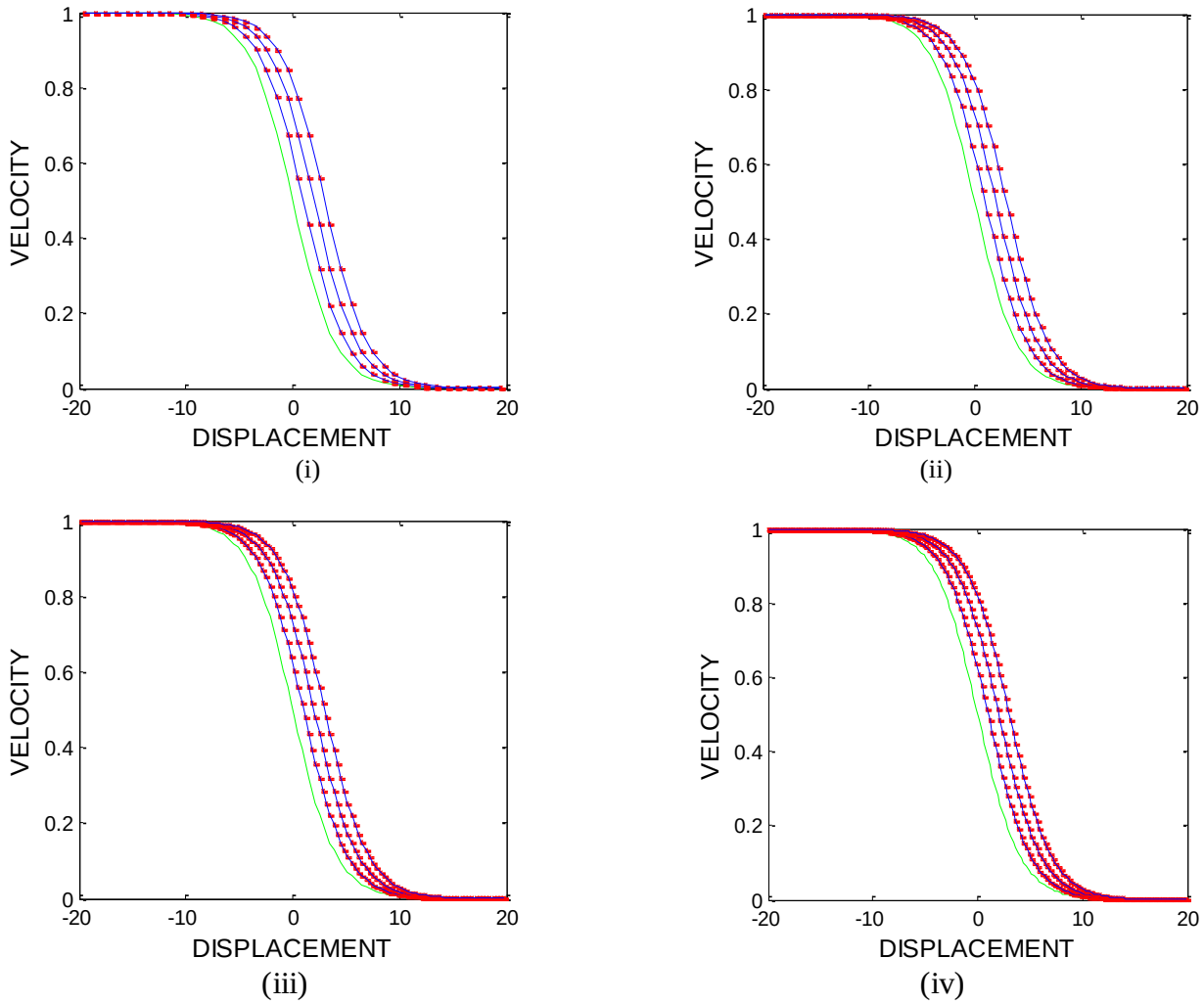


Figure 4. Comparison between NIM (asterix) and analytical solution (blue line) (dt = 0.001, T = 6, δ = 1, for (i) 2a = 1, (ii) 2a = 0.5, (iii) 2a = 0.33, (iv) 2a = 0.25

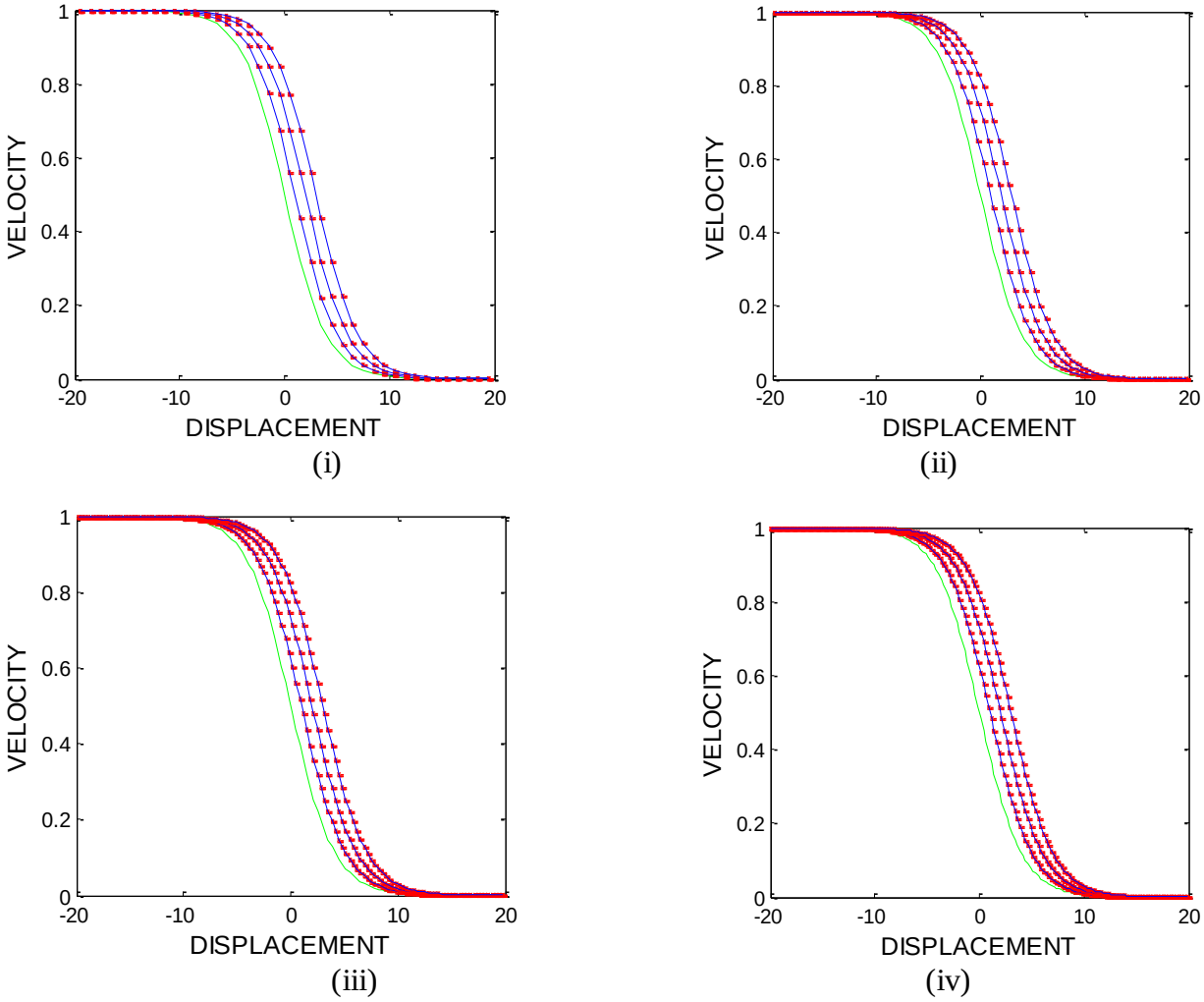
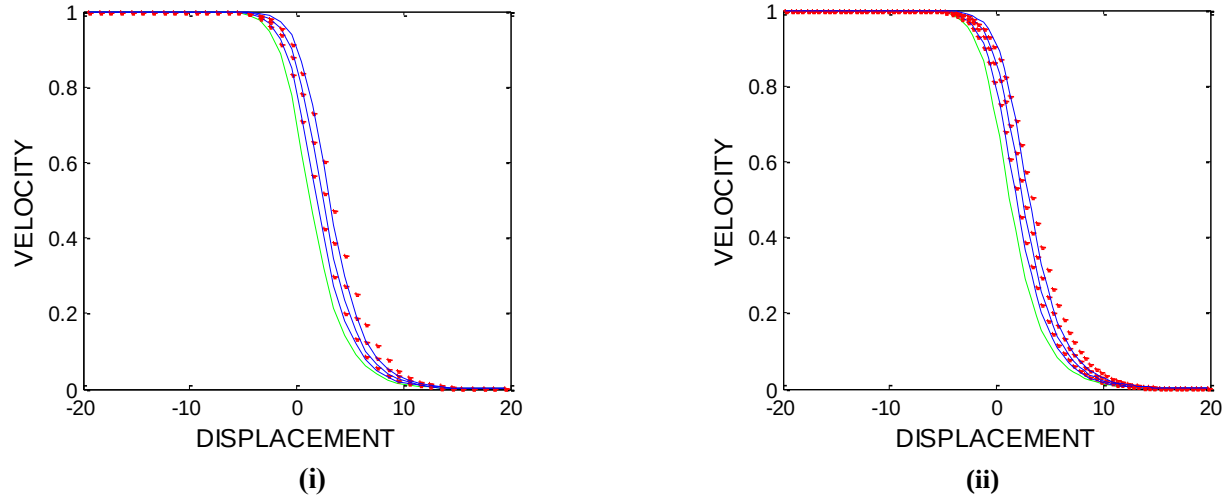
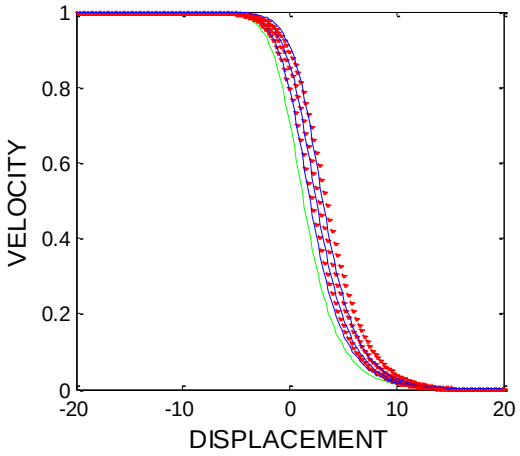
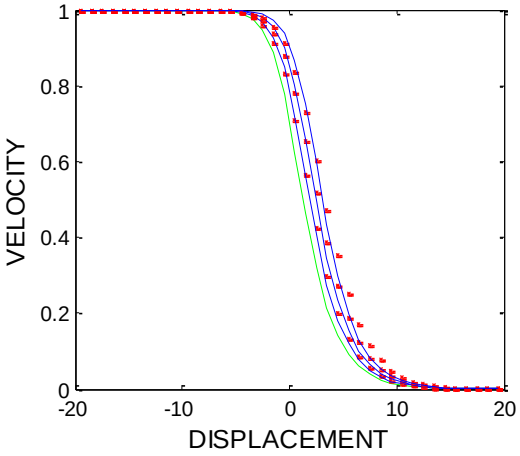


Figure 5. Comparison between NIM (asterix) and analytical solution (blue line) (dt = 0.01, T = 6, δ = 1, for (i) 2a = 1, (ii) 2a = 0.5, (iii) 2a = 0.33, (iv) 2a = 0.25.





(iii)



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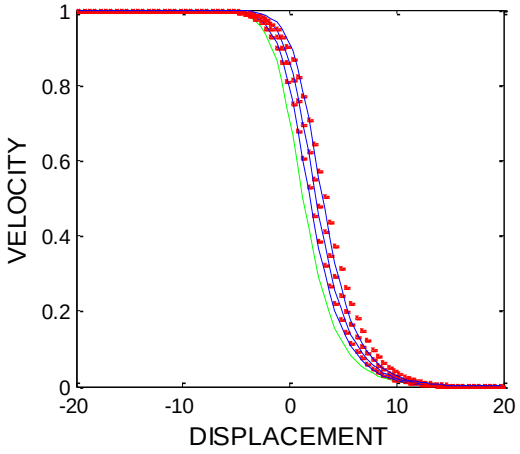

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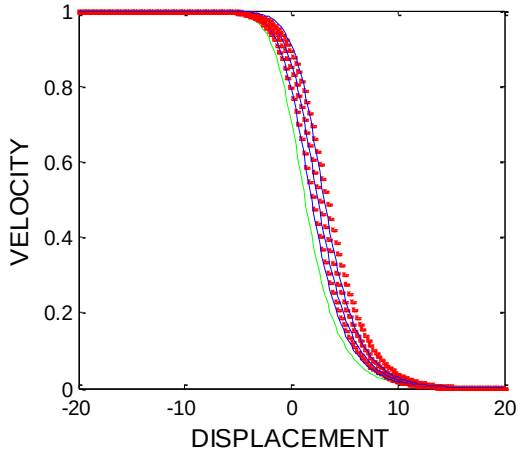

(vi)

Figure 6. Comparison between NIM (asterix) and analytical solution (blue line), T = 6, for (i) 2a = 1, dt = 0.001 and δ = 1, (ii) 2a = 0.5, dt = 0.001 and δ = 1, (iii) 2a = 0.25, dt = 0.001 and δ = 1, (iv) 2a = 1, dt = 0.01 and δ = 2, (v) 2a = 0.5, dt = 0.01 and δ = 2, (vi) 2a = 0.25, dt = 0.01 and δ = 2.

From the Figures 1 to 6 reporting comparison of numerical results with analytical solution, it can be observed that NIM is capable of obtaining better results with coarse nodes. For δ = 1 both the numerical as well as analytical match very well but for δ =2 some error in results is found. These errors in the solutions for δ = 2 are due to higher nonlinearity in the equations. Further norm of absolute errors of NIM compared to analytical solutions for different grid size are calculated and plotted in log-log scale. It can observed from the Figure 7 that the scheme is second order as graph is straight line with a slope of 2.

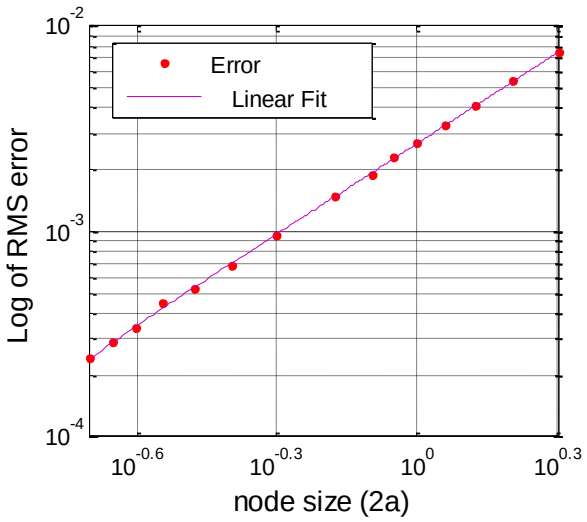

Figure 7. Log of RMS error vs log of node sizes

Further, the CPU running time for δ = 1 and δ = 2 are plotted with various node sizes and are given in Figure 8. To the best of author's knowledge no such analysis is found in literature. Here the CPU running time is calculated up to one integral time step.

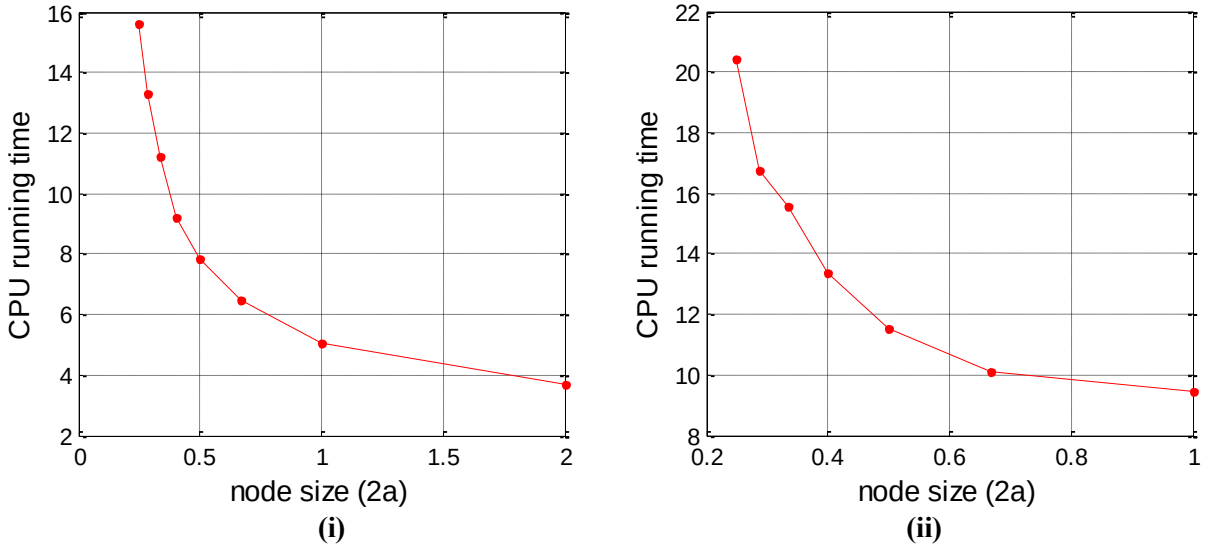

Figure 8. Plot for CPU running time vs grid size for (i) δ = 1, (ii) δ = 2

Moreover, the comparison of NIM with Variational Iteration Method (VIM) [13], Adomian Decomposition Method (ADM) [11] and Differential Quadrature Method (DQM) [14] are done for different parameters reported here. The comparisons are listed in Table 1 to 5. It is found that the NIM is much more accurate than VIM as well as ADM. In the reference, Adomian decomposition method is used to approximate the differential equation by infinite convergent series. On the other hand, DQM is fully based on the higher degree approximation of spatial derivatives whereas VIM is started with trial function and with iterations the order of approximation is increased.

Table 1. Absolute error between of MNIM with ADM and VIM (α = 1, γ = 0.001, β = 1, δ = 1, dt = 0.0001)

α = 1	**β = 1**	**γ = 0.001**	**δ = 1**	**dt = 0.0001**
x	**t**	**ADM** [11]	**VIM** [13]	**NIM**
0.1	**0.05**	1.94 E-07	1.87 E-08	2.05E-08
	0.1	3.87 E-07	3.75 E-08	4.93E-08
	1	3.88 E-06	3.75 E-07	7.10E-07
0.5	**0.05**	1.94 E-07	1.87 E-08	3.33E-09
	0.1	3.87 E-07	3.75 E-08	1.82E-08
	1	3.88 E-06	3.75 E-07	6.57E-07
0.9	**0.05**	1.94 E-07	1.87 E-08	2.01E-08
	0.1	3.87 E-07	3.75 E-08	4.89E-08
	1	3.88 E-06	3.75 E-07	7.10E-07

Table 2. Absolute error between MNIM and ADM (α = 1, γ = 0.001, β = 0, δ = 1, dt = 0.0001)

α = 1	**β = 0**	**γ = 0.001**	**δ = 1, dt = 0.0001**
x	**t**	**Adomian** [11]	**NIM**
0.1	**0.5**	6.34E-08	6.20E-11
	1	2.03E-06	3.10E-11
	2	6.43E-05	2.19E-10
0.5	**0.5**	5.67E-08	6.20E-11
	1	1.85E-06	3.10E-11
	2	6.07E-05	2.19E-10
0.9	**0.5**	4.13E-08	6.20E-11
	1	1.38E-06	3.10E-11
	2	4.75E-05	2.19E-10

Table 3. Absolute error between MNIM and ADM (α = 0.001, γ = 0.001, β = 0.001, δ = 1, dt = 0.0001)

α = 0.001	**β = 0.001**	**γ = 0.001**	**δ = 1, dt = 0.0001**
x	**t**	**Adomian** [11]	**NIM**
0.1	**0.005**	9.69E-06	5.70E-12
	0.001	1.94E-06	6.70E-12
	0.01	1.94E-05	5.60E-12
0.5	**0.005**	9.69E-06	4.60E-12
	0.001	1.94E-06	6.60E-12
	0.01	1.94E-05	2.10E-12
0.9	**0.005**	9.69E-06	5.50E-12
	0.001	1.94E-06	6.65E-12
	0.01	1.94E-05	5.20E-12

Table 4. Absolute error between MNIM and DQM (α = 1, γ = 0.001, β = 1, δ = 1, dt = 0.0001)

α = 1	**β = 1**	**γ = 0.001**	**δ = 1, dt = 0.0001**
x	**t**	**DQM** [14]	**NIM**
X3	**0.2**	6.84E-09	1.08E-07
	0.6	7.73E-09	4.03E-07
	0.8	7.75E-09	5.51E-07
X7	**0.2**	3.64E-08	7.34E-08
	0.6	4.23E-08	3.60E-07
	0.8	4.24E-08	5.10E-07
X13	**0.2**	1.42E-08	9.53E-08
	0.6	1.62E-08	3.86E-07
	0.8	1.62E-08	5.37E-07

Table 5. Absolute error between MNIM and DQM (α = 0.1, γ = 0.0001, β = 0.001, δ = 1, dt = 0.0001)

x	**t**	**DQM** [14]	**NIM**
X3	0.3	2.32E-09	1.22E-08
	0.5	2.41E-09	2.67E-08
	0.9	2.43E-09	5.67E-08
X7	0.3	6.58E-09	7.37E-09
	0.5	6.90E-09	2.17E-08
	0.9	6.95E-09	5.16E-08
X13	0.3	3.70E-09	1.02E-08
	0.5	3.86E-09	2.47E-08
	0.9	3.88E-09	5.46E-08

For comparison, all the parameters are kept same as in the mentioned references. It is clearly seen from the tables listed above that MNIM is able to approximate the solution with high accuracy, as compared to ADM and VIM. However, there is slightly higher error in NIM with respect to DQM, but the development process as well as implementation of NIM is quite simple and straightforward as compared to DQM.

5 Conclusion

In the present work, modified NIM has been developed for the Burgers-Huxley equation. The consistency of the scheme has been shown by comparing the solutions obtained by the present scheme with the analytical solutions for various parameters. It can be seen that the developed scheme gives more accurate solution with coarse grid. Moreover, comparison of the present scheme have been carried out using ADM, VIM, and DQM. It is observed that NIM is significantly more accurate than the methods used earlier to solve these equations. However, DQM is a bit more accurate, but it is significantly more complex in development as well as implementation. Moreover, this scheme can be further sped up by using appropriate preconditioner and more accuracy can be achieved by use of different approximation for nonlinear source term of the equation.

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