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Hydrodynamic dispersion in Darcy-non Darcy porous medium filled by a nanofluid, in Istanbul, Turkey, 2016

A.M Bouaziz[1], M.N Bouaziz[1], S. Hanini[1]

[1]Biomaterials and Transport Phenomena Laboratory,
Dr Yahia Fares University, Medea 26000, Algeria.

Corresponding author: mn_bouaziz@email.com

Abstract: The hydrodynamic dispersion and the pore diameter are the origin of the thermal and solutal dispersion on the boundary layer between a vertical plate immersed into a Darcy or non-Darcy saturated porous medium filled by a nanofluid. The similarity transformations are involved and the governing system of nonlinear partial differential equations is converted into a set of nonlinear ordinary differential equations. Results are displayed graphically to illustrate the influence of dispersion parameters on the velocity and the volume fraction of the nanoparticles profiles. Substantial effects are found and both skin friction and rate of heat and mass coefficients are affected.

Keywords: Hydrodyamic dispersion, nanofluid, non-Darcy porous medium, skin friction coefficient.

1 Introduction

One way to increase heat transfer in the porous medium is to employ a nanofluid [1]. A few percent of nanoparticles made in metals can improve the thermal conductivity of the nanofluid in the range of 15 – 40 % over the base fluid. It is also understood that typical augmentations in the range of 10 – 50 %, even up to 100 % are due to the micro-convection which was attributed to the Brownian motion. The complexity of the flow through a porous medium is described by the Darcy-Forchheimer law when inertial effects dominate. Additionally, when coupled heat and mass transfer occur, and inertial effects are prevalent, the thermal and solutal dispersion becomes significant. The dispersion or hydrodynamic dispersion can be caused by flow and geometrical obstruction as well [2]. It is generally attributed to the velocity distribution in each variable pore. Another way to increase heat transfer is to employ the porous medium with nanofluid. The complexity of the flow through a porous medium is described by the Darcy-Forchheimer law when inertial effects dominate. Additionally, when coupled heat and mass transfer occur, and inertial effects are prevalent, the thermal and solutal dispersion becomes significant. These effects were also observed with a certain order in vigorous natural convection flows.The dispersion or hydrodynamic dispersion can be caused by flow and geometrical obstruction as well [3]. It is generally attributed to the velocity distribution in each variable pore.
It should emphasize that the thermal dispersion due to the velocities of the base-fluid, and the thermal dispersion generated by the fine particles or nanoparticle dispersion are not similar.
Based on the literature reviewed, it is clear that the role of the dispersion of the nanoparticle is still unclear. On the other side, it was well established that the dispersion caused by the flow was correlated to the longitudinal and transversal thermal or solutal transfer.
Precisely, the hydrodynamic dispersion refers to a solid-liquid mixture in a porous medium of pore diameter within 20-800 μm.

Kuznetsov and Nield studied the general problem passing a vertical plate in thermal diffusive case [4] and in double diffusive case [5]. They revisited their model, including the passive boundary condition [6]. Nield and Kusnetsov in [7-10] examined the diffusive, the double diffusive boundary layer and the revised model in a porous medium.
Based on the cited papers, the objective of the present contribution is to highlight the double dispersion effect on a double diffusive boundary layer, in a non-Darcy porous medium saturated by a nanofluid, which occurs particularly in the strong natural convection. The passive nanoparticle boundary condition is taking into account and both the non-isothermal and convective boundary cases are examined.

2 Problem Formulation

Consider the steady laminar free convection flow along a vertical plate immersed in a homogeneous and isotropic porous medium with porosity ε. The Forshheimer- extended Darcy's equation of motion is assumed to describe the higher flow. The nanofluid involved is composed by a mixture of a base fluid which is completed additionally by a volume fraction ϕ of the solid nanoparticles. Only the two fundamental mechanisms, namely the Brownian motion and the thermophoresis effect are taking into account in modelling the behaviour of the nanofluid [11]. For the volume fraction of the nanoparticles, an additional equation is included to the mathematical model. The wall is maintained at constant temperature (case 1: isothermal) but often the convective boundary condition (case 2: non-isothermal) is recognized more appropriate.

The basic model developed in the work of Nield and Kuznetsov [8] is used. The system of equations written in natural form is

$$\nabla.\mathbf{v} = 0$$

$$0 = -\nabla p - \frac{\mu}{K}\mathbf{v} - \rho_{f\infty}\frac{c_f}{\sqrt{K}}\mathbf{v}|\mathbf{v}| + \left[\phi\rho_p + (1-\phi)\rho_f\left(1-\beta_T(T-T_\infty)-\beta_C(C-C_\infty)\right)\right]\boldsymbol{g}$$

$$(\rho c)_f\mathbf{v}.\nabla T = k_m\nabla^2 T + \varepsilon(\rho c)_p\left[D_B\nabla\phi.\nabla T + \left(\frac{D_T}{T_\infty}\right)\nabla T.\nabla T\right]$$

$$\frac{1}{\varepsilon}\mathbf{v}.\nabla C = D_S\nabla^2 C$$

$$\frac{1}{\varepsilon}\mathbf{v}.\nabla\phi = D_B\nabla^2\phi + \left(\frac{D_T}{T_\infty}\right)\nabla^2 T$$

These equations are subject to the following boundary conditions

$$at\ y = 0, v = 0, T = T_w = T_f\ (\textbf{case 1}) \qquad or \qquad -k_m\frac{\partial T}{\partial y} = h\left(T_f - T\right)(\textbf{case 2}),$$

$$C = C_w,\ D_B\frac{\partial\phi}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial T}{\partial y} = 0$$

$$as\ y \to \infty,\ u = v = 0,\ T \to T_\infty,\ C \to C_\infty,\ \phi \to \phi_\infty$$

The porous medium is characterized by the porosity ε, the permeability K and the empirical constant associated with inertia effect term c_f .The gravitational acceleration vector is indicated by **g.** β_T and β_C are the volumetric thermal expansion and the equivalent solutal coefficient of the base fluid. The letters ρ and p are the density and pressure of the nanofluid. Subscript p represents the particle and f the base fluid. The coefficients D and μ are the diffusion coefficient and the viscosity of the nanofluid.

Subscripts B, T, and S referred to the Brownian motion, the thermophoretic effect and the solutal diffusivity of the porous medium, respectively. The effective thermal conductivity is k_m for the porous medium and heat capacity is (ρc). h is the heat transfer coefficient between the plate and the hot fluid.

Following the works for porous medium, dimensionless quantities are introduced to obtain the similarity solutions

$$\eta = \frac{y}{x} {Ra_x}^{1/2}, f(\eta) = \frac{\psi}{\alpha_m\, {Ra_x}^{1/2}} \quad \left(u = \frac{\partial\psi}{\partial y} \text{ and } v = -\frac{\partial\psi}{\partial x}\right),$$

$$\theta = \frac{T-T_\infty}{T_f-T_\infty}, \gamma = \frac{C-C_\infty}{C_w-C_\infty}, s = \frac{\phi-\phi_\infty}{\phi_w-\phi_\infty}$$

The local Rayleigh number is then introduced and pertinently defined by

$$Ra_x = \frac{(1-\phi_\infty)\rho_{f\infty}\beta_T(T_f-T_\infty)gKx}{\mu\alpha_m}$$

The complete transformed dimensionless partial differential equations N-S energy, concentration and micro-convection accompanied with the corresponding boundary conditions are:

$$f''[1 + 2F_0 Ra_d f'] = \theta' + N_c\gamma' - N_r s';$$

$$\theta''[1+\delta Ra_d f'] = -\frac{1}{2}f\theta' - N_b s'\theta' - N_t(\theta')^2 - \delta Ra_d \theta' f'';$$

$$\gamma''[1+\xi Ra_d f'] = -\frac{1}{2}L_e f\gamma' - \xi L_e Ra_d \gamma' f'';$$

$$s'' = -\frac{1}{2}L_n f s' - \frac{N_t}{N_b}\theta''$$

$$\text{at } \eta = 0,\ f = 0,\quad \theta = 1 \ (\textbf{case 1}) \quad \text{or} \quad \theta' + Bi(1-\theta) = 0 \ (\textbf{case 2}),$$
$$\gamma = 1,\quad N_b s' + N_t\theta' = 0 \quad \text{as } \eta \to \infty,\ f' = 0,\ \theta \to 0,\ \gamma \to 0,\ s \to 0$$

F_0 is a parameter that represents the structure of the porous medium. It combines the inertia effect and the diffusivity of the saturated porous medium. Ra_d, is the preceding Rayleigh number which is now expressed with d, the pore diameter. N_c, N_r, N_b, and N_t are the regular double-diffusive buoyancy ratio, the nanofluid buoyancy ratio, the Brownian motion parameter and the thermophoresis parameter respectively. L_e denotes the usual Lewis number, L_n is a nanofluid Lewis number. The new constant of solutal dispersion ξ is set equal to $\varepsilon\sigma$. In the convective boundary condition case, the interaction between the plate and the hot fluid is represented by the Biot number Bi.

For practical applications, the physical quantities of most interest are the wall heat flux q_w and the wall mass flux q'_w and the corresponding local dimensionless physical quantities may be quantified by

$$Nu_x {Ra_x}^{-1/2} = -[1 + \delta Ra_d f'(0)]\theta'(0)$$
$$Sh_x {Ra_x}^{-1/2} = -[1 + \xi Ra_d f'(0)]\gamma'(0)$$

3 Results

After some validation the above system is solved for the two cases. Some results are visualized in figure 1, figure 2 and table 1.

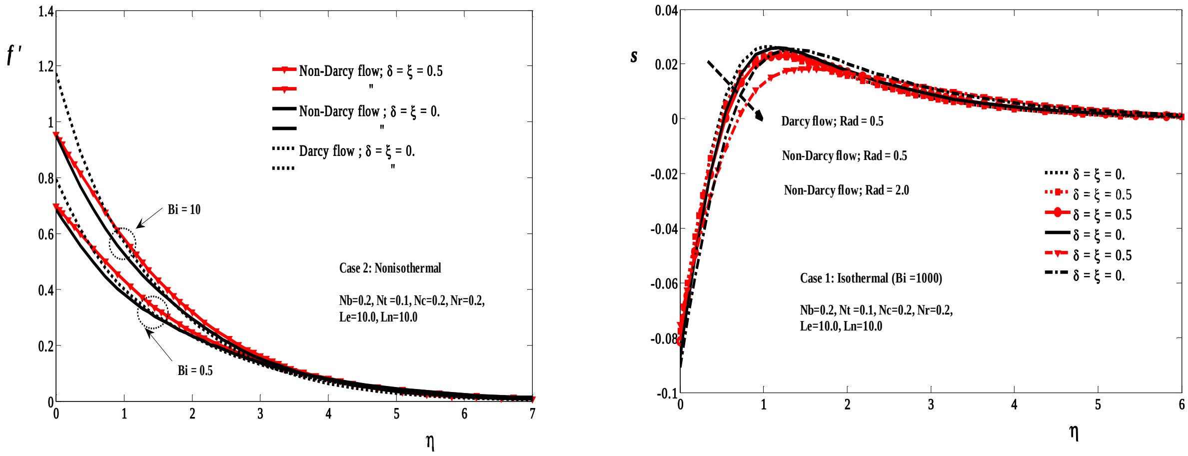

Figure 1: The effect of the double dispersion and the Non-Darcy flow on the velocity and on the volume fraction profiles.

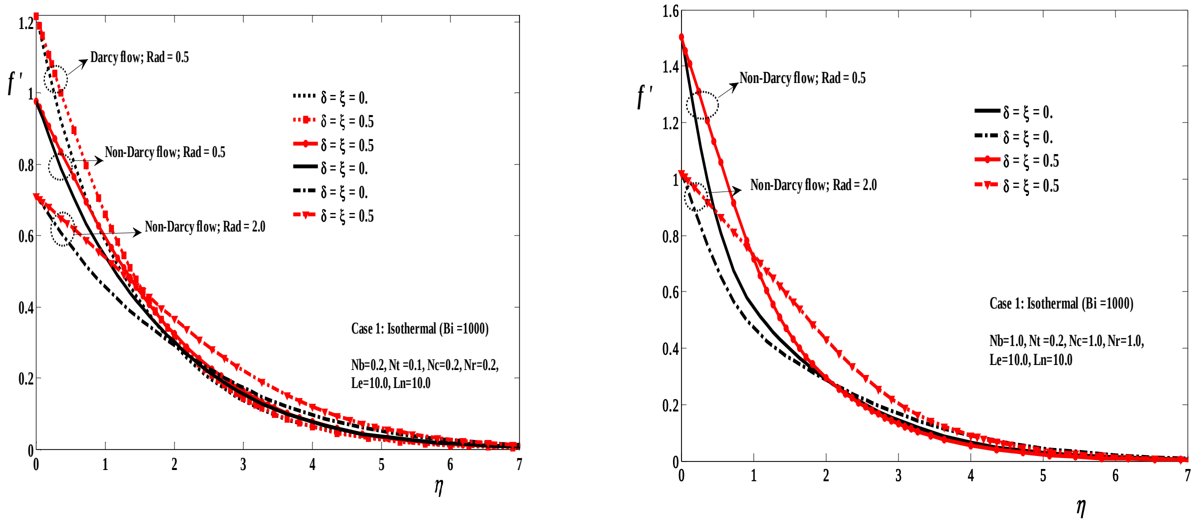

Figure.2. The effect of the double dispersion and the Non-Darcy flow on the velocity profile. Bi = 1000 and weak and strong values [N_b; N_t; N_c; N_r] for the nanofluid.

Table 1. Values of $Nu_x Ra_x^{-1/2}$ *and* $Sh_x Ra_x^{-1/2}$ for selected values of *Bi*, Ra_d and *Fo* with (N_b=1.0, N_t =0.2, N_c=1.0, N_r=1.0, L_e=10.0, L_n=10.0).

Bi	Ra_d	*Fo*	$Nu_x Ra_x^{-1/2}$		$Sh_x Ra_x^{-1/2}$	
			$\delta=\xi=0$	$\delta=\xi=0.5$	$\delta=\xi=0$	$\delta=\xi=0.5$
1000	0.5	0	0.451253	0.581987	2.234237	1.166484
		0.5	0.409325	0.498100	1.952215	1.089621
	2.0	0.5	0.355067	0.534220	1.636827	0.847836
10	0.5	0	0.427590	0.477807	2.205241	1.156426
		0.5	0.390453	0.477121	1.933264	1.088500
	2.0	0.5	0.341270	0.517677	1.625027	0.843166
0.5	0.5	0	0.219472	0.282625	1.913743	1.046890
		0.5	0.211038	0.264826	1.726015	0.999676
	2.0	0.5	0.197463	0.323072	1.485511	0.782244

4 Conclusion

The effects of the double dispersion on the double diffusive convective boundary layer developed between a vertical plate that is immersed into a non-Darcy saturated porous medium with a nanofluid are examined. The similarity transformations are involved and the governing system of nonlinear partial differential equations is converted into a set of nonlinear ordinary differential equations. Numerical results are presented and we can conclude the followings from our investigations:

- The double dispersion has a strong effect on the heat and mass convective transfers. This effect is more pronounced for a nanofluid than for a clear fluid. This is also true for mass transfer flux than for heat transfer flux, in the case $L_e = L_n = 10$.
- Working with a nanofluid inside a non-Darcy porous medium leads to modifying the velocity, the temperature and the concentration of the species mass profiles, which in turn affect the rates of heat and mass, when the double dispersion acts.
- It is found that the rate of heat transfer increases and the mass transfer decreases strongly with the double dispersion.

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